8. Vertical stress below applied load

(1 th + 1 Num)

8.1 Concept of stress distribution

Stresses are induced in soil mass due to;
i) Self wt. of soil layers (geostatic stress)
ii) Added load from structures such as buildings, bridges, dams, embankment etc.

Geostatic stress can be divided into;
i) Vertical geostatic stress (σ_v)

$$\boxed{\sigma_v = \gamma H}$$

where;

$\gamma \rightarrow$ unit wt. of soil

$H \rightarrow$ ht from ground surface

Vertical geostatic stress increases with increase in depth from ground surface.

ii) Horizontal geostatic stress / Radial (σ_H / σ_R)

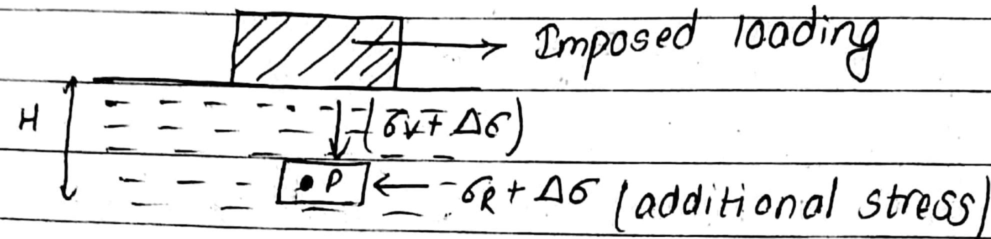
$$\boxed{\sigma_R = K_o \times \sigma_v}$$

where;

K_o = coeff. of radial stress = $1 - \sin\phi$

ϕ = angle of internal friction

Note:- If there are imposed structural loadings on the soil, the resultant stress may be obtained by adding algebrically the stress due to self wt and stress transmitted due to structural loading.



so, when a load is applied to the soil surface, the vertical stress within the soil mass increases. The vertical stresses at any point in a soil mass due to external vertical loadings are of great significance in the predection of settlements of buildings, bridges, embankments etc.

8.2 Boussinesq's and westergaard theory

Boussinesq theory

Assumptions:-

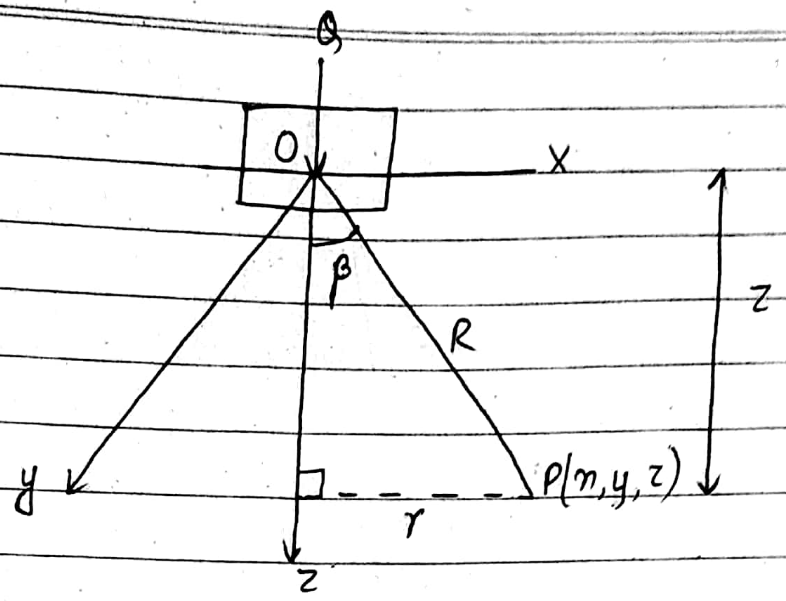
i) The soil mass is linearly elastic medium i.e the modulus of elasticity (E) is constant throughout the considered area.

2) The soil is homogenous & isotropic i.e it has identical properties in all directions.

3) The soil is initially unstressed.

4) The self wt. of soil is ignored.

5) The soil mass is semi-infinite i.e it extends infinitely in all directions below the ground surface.



Let a vertical point load Q be acting at the soil surface at point O. Let P be the point in the soil mass having co-ordinates (x, y, z)

Using logarithmic stress function;

Boussinesq gave the value of radial stress as;

$$\sigma_R = \frac{3Q}{2\pi R^2} \times \cos\beta$$

$$R = \sqrt{x^2 + y^2 + z^2}$$

let; $r^2 = x^2 + y^2$

$$R = \sqrt{r^2 + z^2}$$

Now;

The value of vertical stress (σ_v) is given as;

$$\sigma_v = \sigma_R \times \cos^2\beta$$

$$= \frac{3Q}{2\pi R^2} \cos^2\beta$$

$$= \frac{3Q}{2\pi R^2} \left(\frac{z}{R}\right)^3 \quad \text{[from fig]}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{R^2} \times \frac{z^3}{R^3}$$

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$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \times \frac{z^5}{R^5}$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \left[\frac{(z^2)^{5/2}}{(r^2+z^2)^{5/2}} \right]$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \left[\frac{1}{\left(\frac{r^2}{z^2} + \frac{z^2}{z^2}\right)^{5/2}} \right]$$

$$= \frac{3Q}{2\pi} \times \frac{1}{z^2} \left(\frac{1}{\frac{r^2}{z^2} + 1} \right)^{5/2}$$

$$= \frac{Q}{z^2} \times \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

Point load

$$= \frac{Q}{z^2} \times I_B$$

where;

$$I_B = \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

= Boussinesq influence factor

where;

r = radial distance from the axis of load

$$\therefore \boxed{\sigma_v = \frac{Q}{z^2} \times I_B}$$

Putting $r = 0$; σ_v is maximum;

$$\sigma_v = \frac{Q}{z^2} \times \frac{3}{2\pi}\left[\frac{1}{1+0}\right)$$

$$\boxed{\sigma_{v\,max} = 0.477\,\frac{Q}{z^2}}$$

N.1 A water tower weighing 15000 KN is to be considered as a concentrated load acting on the ground surface. Compute the vertical stress at a depth of 8m below the surface. Also compute the vertical stress at a distance of 7m away from centre of water tower.

Sol^n

Case I :-

load of water tower $(Q) = 15000$ KN

vertical depth from ground surface $(z) = 8$m

$r = 0$ (below the surface)

Now; $\sigma_v = \frac{Q}{z^2} \times I_B$

$$I_B = \frac{3}{2\pi}\left(\frac{1}{1+0}\right) = 0.477$$

$$\sigma_v = \frac{15000 \times 10^3}{8^2} \times 0.477$$

$$= 111.79 \text{ KN/m}^2$$

Case II :-

$r = 7$m

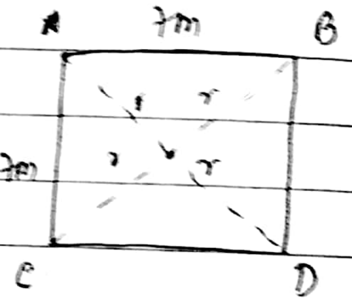
$$I_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{7}{8}\right)^2}\right]^{5/2} = 0.115$$

$$\sigma_v = \frac{15000\times10^3}{8^2}\times 0.115$$

$$= 27.01\ kN/m^2$$

2018 fall An overhead water tank has a wt. of 1600 kN is supported on a tower with 4 legs. The legs rest on a pier located at the corner of square with 7m length. Determine the increase in vertical stress at a depth 4.5m below the centre of square.

Soln



total wt of tower = 1600 kN

wt. on each legs = $\frac{1600}{4}$

$= 400$ kN

vertical depth $(z) = 4.5$ m

$r = \sqrt{3.5^2 + 3.5^2}$

$= 4.94$ m

Now;

σ_v for one leg $= \frac{Q_A \times I_B}{z^2}$

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$$I_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$= \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{4.94}{4.5}\right)^2}\right]^{5/2}$$

$$= 0.066$$

$$\sigma_{v_A} = \frac{400}{4.5^2} \times 0.066$$

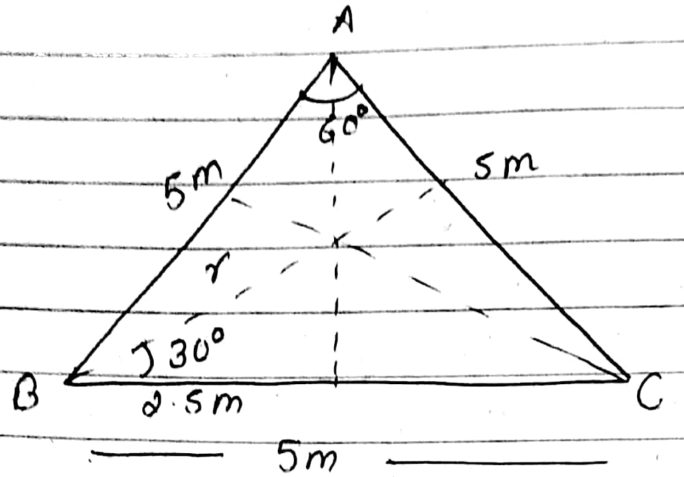
$$= 1.306\ KN/m^2$$

Resultant vertical stress; $= 4 \times 1.306$

$= 5.22\ KN/m^2$.

2010
2012
Spring

The plan of a 3 leged tower forms an equilateral triangle of side 5m. If the total wt. of tower is 600 kN & equally carried out by all the legs. compute the stress increase in the soil by the tower at the depth of 4m below the centre of equilateral triangle.



Sol^n

$$\cos 30^\circ = \frac{2.5}{r}$$

$$r = 2.88m$$

$$\sigma_v = \frac{Q_A}{z^2} \times I_B$$

$$I_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$= \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{2.88}{4}\right)^2}\right]^{5/2}$$

$$= 0.168$$

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$$\sigma v_A = \frac{\left(\frac{600}{3}\right)}{4^2} \times 0.168$$

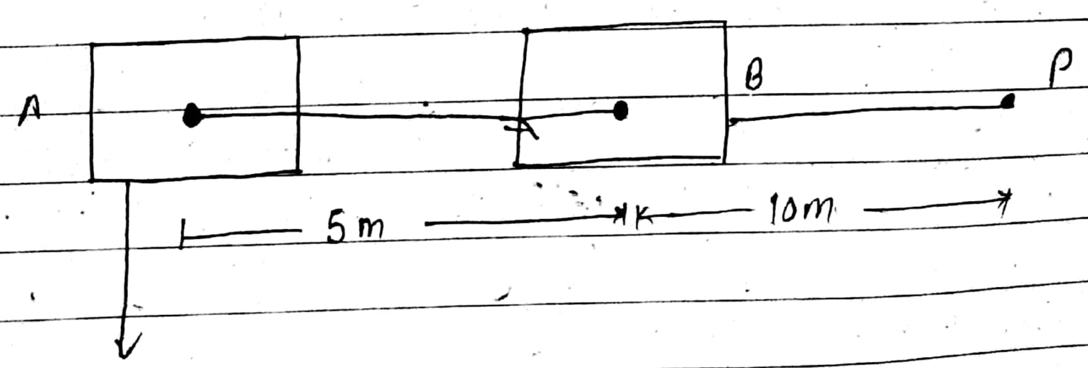
$$= 2.100 \ KN/m^2$$

Resultant vertical stress = 3×2.100
$= 6.302 \ KN/m^2$

2008 Fall

Two columns A & B are placed at 5m centre to centre. Through point A, a load of 400 kN is acting & from point B, a load of 240 kN is acting. Calculate the vertical stress due to these loads on a horizontal plane 2m below the ground surface at points;

i) vertically below points A & B
ii) 10 m horizontally away from point B.



i) Stress vertically below A; $r = 0$ $\quad r = 5$

= Stress due to A at A + stress due to B

$$= \frac{Q_A}{z^2} \times I_B + \frac{Q_A}{z^2} \times I_B$$

$$= \frac{240}{2^2} \times \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{5}{2}\right)^2} \right]^{5/2} + \frac{400}{2^2} \times \frac{3}{2\pi}$$

$$= 47.90 \text{ kN/m}^2$$

Stress vertically below B,

$$= \frac{400}{2^2} \times \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{5}{2}\right)^2} \right]^{5/2} + \frac{240}{2^2} \times \frac{3}{2\pi}$$

$$= 28.98 \text{ kN/m}^2$$

ii) vertical stress at 10 m from B $r = 15$ $\quad r = 10$

= stress at P due to A + stress due to B

$$= \frac{400}{2^2} \times \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{15}{2}\right)^2} \right]^{5/2} + \frac{240}{2^2} \times \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{10}{2}\right)^2} \right]^{5/2}$$

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$= 1.92 \times 10^{-3} + 8.311 \times 10^{-3}$

$= 0.010\ kN/m^2$.

2008 Spring
2015 Fall

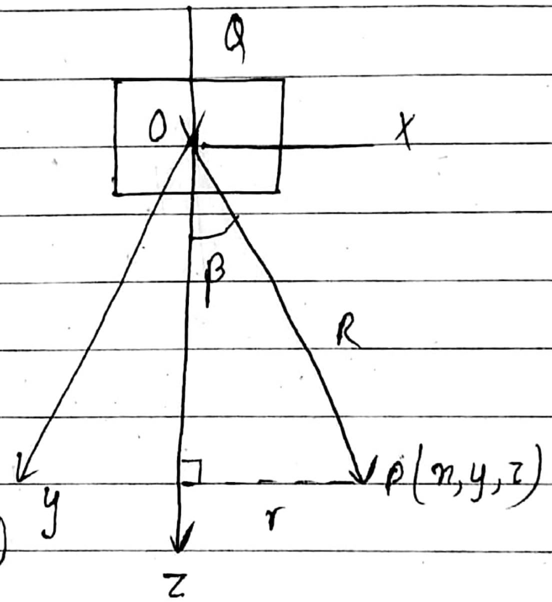
A concentrated point load of 250 kN acts at the ground surface. Find the intensity of the vertical pressure at depth 5m below the ground surface at the point on the axis of loading using Boussinesq. What will be the difference in vertical pressure at the same point if the load is shifted to a distance of 2m from it's original position horizontally.

Westergaard theory

Assumptions:-

i) The soil mass is elastic in nature.
ii) The soil mass is anisotropic (stratified).
iii) The soil mass is semi-infinite.
iv) The soil mass is previously unstressed.
v) The self weight of soil is completely ignored.

According to westergaard, The vertical stress at a point 'P' at a depth 'z' is given by;



$$\sigma_v = \frac{\frac{C}{2\pi}}{\left[C^2 + \left(\frac{r}{z}\right)^2\right]^{3/2}} \times \frac{Q}{z^2} \quad \text{—} \; (1)$$

where; the value of 'c' depends upon poisson's ratio (ν)

$$C = \sqrt{\frac{1-2\nu}{2-2\nu}}$$

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for elastic material, the value of poisson's ratio varies from 0 to 0.5

for practical purpose;

taking $\gamma = 0$;

$$C = \sqrt{\frac{1-2\times 0}{2-2\times 0}} = \frac{1}{\sqrt{2}}$$

Putting the value of C in eq^n 1

$$\sigma_v = \frac{\frac{1/\sqrt{2}}{2\pi}}{\left[\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{r}{z}\right)^2\right]^{3/2}} \times \frac{Q}{z^2}$$

$$= \frac{Q}{\pi z^2} \times \frac{(1/2)^{3/2}}{\left[\frac{1}{2} + \left(\frac{r}{z}\right)^2\right]^{3/2}}$$

$$= \frac{Q}{\pi z^2}\left[\frac{1}{\frac{1/2}{1/2} + \frac{(r/z)^2}{1/2}}\right]^{3/2}$$

$$= \frac{Q}{\pi z^2} \times \left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}$$

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$$= \frac{Q}{z^2} \times I_w$$

where; $I_w = \frac{1}{\pi}\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2}$

= westergaard influence factor

for max σ_v; $r = 0$

$$I_w = \frac{1}{\pi}$$

$$\sigma_{v\,max} = \frac{Q}{\pi z^2}$$

Comparison between Boussinesqs and westergaard analysis :-

1) Westergaard theory is used in stratified soil (anisotropic) while Boussinesqs is used in homogenous (isotropic) soil.

2) The vertical stress given by Boussinesqs formula under the point load is higher than the vertical stress given by westergaard.

$$(\sigma_v)\ \text{Boussinesqs} > (\sigma_v)\ \text{westergaard}$$

$$\frac{3}{2}\frac{Q}{\pi z^2} > \frac{Q}{\pi z^2}$$

$$\therefore \sigma_v\ \text{Boussinesqs} = 1.5\,(\sigma_v)\ \text{westergaard}$$

iii) Since vertical ~~stan~~ stress by Boussinesqs is always higher than that of westergaard under the point load i.e at $r=0$ due to consolidation settlement becomes higher and hence Boussinesqs theory gives conservative value as a result it is widely used.

iv) If the value of r/z is less than or equal to 1.5 then the vertical stress by Boussinesq's is always higher than vertical stress given by westergaard. But if $r/z > 1.5$ then σ_v given by westergaard is higher than that of Boussinesqs.

8.3 Approximate method of vertical stress distribution

a) Equivalent point load method
b) 2:1 load distribution method
c) Sixty degree distribution

a) Equivalent point load method:-

It is an approximate method of calculating the vertical stress at any point due to any loaded area. The ~~cont~~ entire area is divided into a number of smaller area units and total distributed load over a unit

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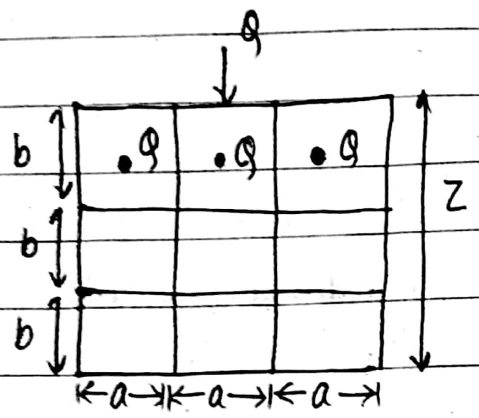
area is replaced by the point load of same magnitude acting at centroid of each area unit.



$a \leq \frac{1}{3} z$

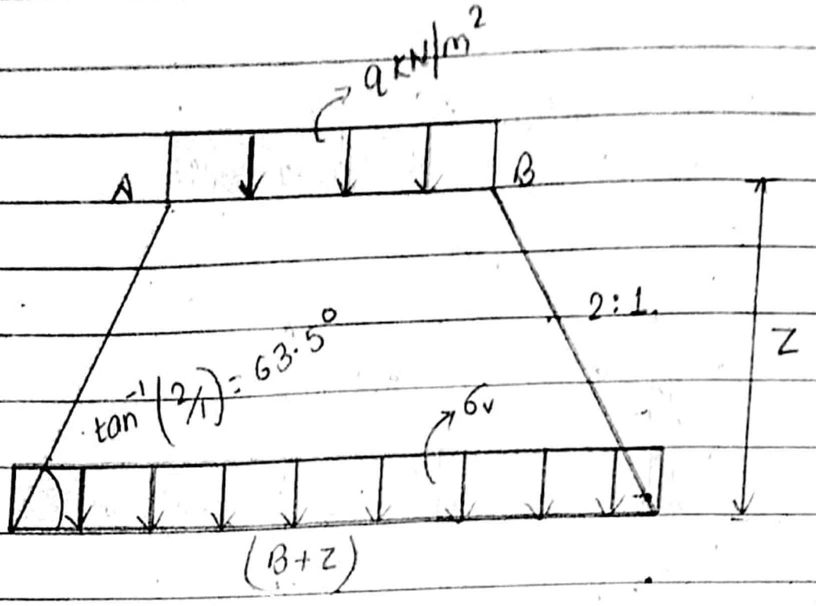
$b \leq \frac{1}{3} z$

$$\sigma_v = \frac{Q_1 \times I_{B_1} + Q_2 \times I_{B_2} + Q_3 \times I_{B_3} + \cdots + Q_n I_{Bn}}{z^2}$$

$$= \frac{1}{z^2} \sum_{i=1}^{n} Q_n (I_B)_n$$

b) 2:1 load distibution method:-

This method is based on the assumption that the load applied at the soil surface is distributed on the soil with approximately 2 (vertical) to 1 (horizontal) spread in the form of a frustrum of a cone as shown:-



vertical stress (σ_v) at a depth z below the soil surface under a different types of surface loading is given by,

i) uniform load on a surface area ($B \times B$)

$$\sigma_v = \frac{qB^2}{(B+z)^2} \quad \left(\frac{\text{force}}{\text{Area}}\right)$$

ii) rectangular area ($B \times L$)

$$\sigma_v = \frac{q \times (B \times L)}{(B+z)(L+z)}$$

iii) circular area (diameter)

$$\sigma_v = \frac{q \times D^2}{(D+z)^2}$$

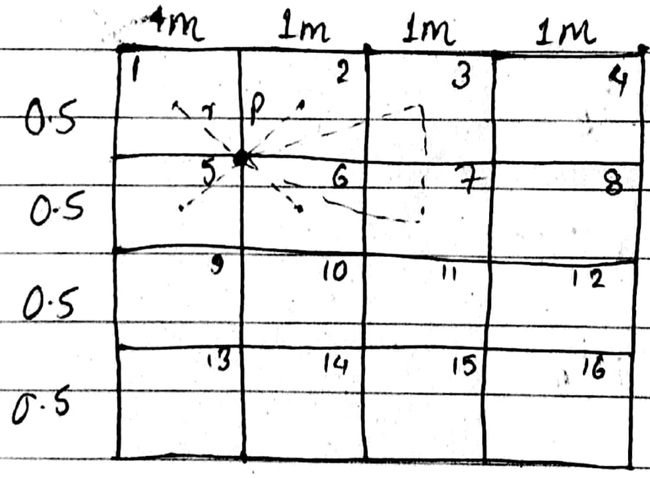
iv) For strip

$$\sigma_v = \frac{q \times B}{B + z}$$

c) 60° distribution:-

This method is similar to 2:1 stress distribution method where the pressure distribution is assumed along the lines making 60° angle with horizontal.

N A rectangular area 4m x 2m is uniformly loaded with a load intensity of 100 kN/m² at the ground surface. Determine the vertical pressure at a depth 3m below a point within the loaded area 1m away from the short edge & 0.5m away from the long edge. Use equivalent point load method.



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load intensity (q) = 100 kN/m^2

Equivalent point load = $\frac{100 \times 4 \times 2}{16}$

= 50 kN

for vertical stress at 1, 2, 5, 6

$r = \sqrt{0.25^2 + 0.5^2}$

= 0.559 m

$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$= \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{0.559}{3}\right)^2}\right]^{5/2}$$

$$= 0.438$$

$$\sigma_v = \frac{4 \times Q}{z^2} \times I_B$$

$$= 4 \times \frac{50}{3^2} \times 0.438$$

$$= 9.742 \text{ kN/m}^2$$

for vertical stress at 3, 7

$$r = \sqrt{0.25^2 + 1.5^2}$$
$$= 1.52\ m$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1 + \left(\frac{1.52}{3}\right)^2}\right]^{5/2}$$

$$= 0.269$$

$$\sigma_v = 2 \times \frac{Q}{z^2} \times I_B$$

$$= 2 \times \frac{50}{3^2} \times 0.269$$

$$= 2.98\ kN/m^2$$

vertical stress at 4 & 8

$$r = \sqrt{0.25^2 + 2.5^2}$$
$$= 2.51$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1 + \left(\frac{2.51}{3}\right)^2}\right]^{5/2}$$

$$= 0.126$$

$$\sigma_v = 2 \times \frac{50}{3^2} \times 0.126$$

$$= 1.4\ kN/m^2$$

vertical stress for 9 & 10

$$r = \sqrt{0.5^2 + 0.75^2}$$
$$= 0.901$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1 + \left(\frac{0.901}{3}\right)^2} \right]^{5/2}$$

$$= 0.38$$

$$\sigma_z = \frac{\cancel{2} \times Q}{z^2} \times I_B$$

$$= \frac{2 \times 50}{3^2} \times 0.38$$

$$= 4.23 \ KN/m^2$$

vertical stress for 13 & 14.

$$r = \sqrt{1.25^2 + 0.5^2}$$
$$= 1.34$$

$$I_B = \frac{3}{2\pi} \left[\frac{1}{1 + \left(\frac{1.34}{3}\right)^2} \right]^{5/2}$$

$$= 0.30$$

$$\sigma_v = 2 \times \frac{Q}{z^2} \times I_B$$

$$= 2 \times \frac{50}{3^2} \times 0.3$$

$$= 3.34 \text{ kN/m}^2$$

For 11

$$r = \sqrt{1.5^2 + \cancel{1.25^2}\, 0.75^2}$$

$$= 1.67$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{1.67}{3}\right)^2}\right]^{5/2}$$

$$= 0.24$$

$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$= \frac{50}{3^2} \times 0.24$$

$$= 1.34 \text{ kN/m}^2$$

For 12; $r = \sqrt{2.5^2 + 0.75^2}$

$$= 2.61$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{2.61}{3}\right)^2}\right]^{5/2}$$

$$= 0.11$$

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$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$= \frac{50}{3^2} \times 0.11$$

$$= 0.61$$

For 15; $r = \sqrt{1.25^2 + 1.5^2} = 1.95$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{1.95}{3}\right)^2}\right]^{5/2}$$

$$= 0.19$$

$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$= \frac{50}{3^2} \times 0.19$$

$$= 1.05 \text{ kN/m}^2$$

For 16; $r = \sqrt{1.25^2 + 2.5^2} = 2.79$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{2.79}{3}\right)^2}\right]^{5/2}$$

$$= 0.1$$

$$\sigma_v = \frac{50}{3^2} \times 0.1$$

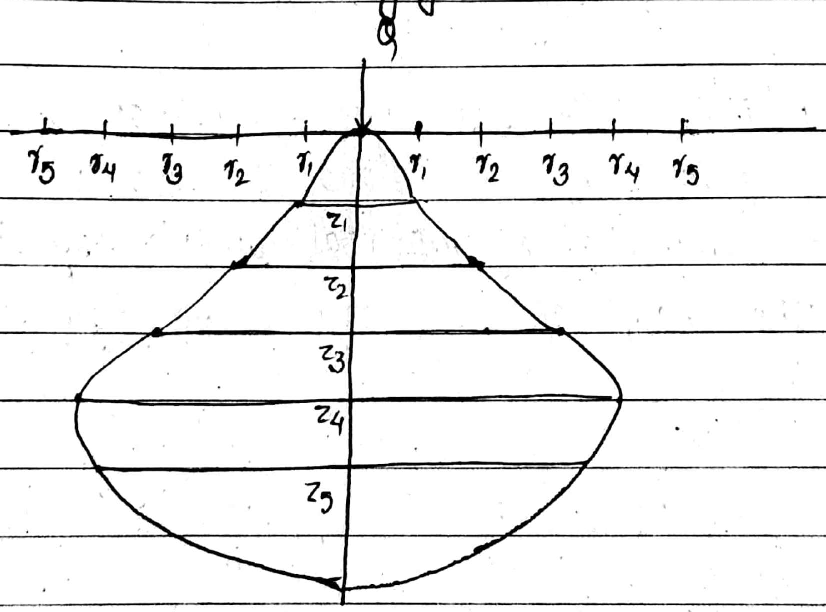
$$= 0.56$$

V.V.V. Imp.

Isobar diagram or, Pressure bulb concept or onion bulb concept:-

An isobar or pressure bulb is a stress contour or a stress line which connects all the point below the ground surface at which the vertical pressure is same. Infact, an isobar is a spatial curved surface and resemble a ~~ball~~ bulb in shape. Thus, stress isobar is also called the bulb of pressure or simply the pressure bulb.

Pressure at points inside the bulb are greater than at a point on the surface of the bulb and pressure at points outside the bulb are so minimum that they are assumed to have negligible stresses.



Procedure to draw an isobar diagram:-

1) Let it be required to plot an isobar for which $\sigma_v = 0.1\,Q$

$$\sigma_v = 0.1\,Q$$

$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$0.1Q = \frac{Q}{z^2} \times I_B$$

$$\boxed{I_B = 0.1z^2}$$

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

2) Assuming various value of z, the corresponding value of I_B can be calculated.
3) for computed values of I_B, the corresponding (r/z) value can be calculated.
4) for assumed value of z, value of r can be calculated from 3.
5) It is obvious that for some value of r on any side of line of action of point load the value of σ_v is same. Hence, the isobar is symmetrical with respect to the axis. So, the other half can be drawn by symmetry.

2013 spring

A point load of 160 KN is applied at the ground surface. Construct a pressure bulb when the stress imposed becomes 10% of the applied load.

Given;

$Q = 160$ KN

$$\sigma_v = \frac{10}{100} \times Q$$

$$\sigma_v = 0.1Q$$

we have;

$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$0.1\cancel{Q} = \frac{\cancel{Q}}{z^2} \times I_B$$

$$\boxed{I_B = 0.1z^2}$$

Now,

$$I_B = \frac{3}{2\pi} \times \left[\frac{1}{1+\left(r/z\right)^2}\right]^{5/2}$$

or; $$0.1z^2 = \frac{3}{2\pi}\left[\frac{z^2}{z^2+r^2}\right]^{5/2}$$

or; $$0.1\cancel{z^2} = \frac{3}{2\pi} \times \frac{z^{\cancel{5}3}}{(r^2+z^2)^{5/2}}$$

or; $$\frac{0.1 \times 2\pi}{3} = \frac{z^3}{(r^2+z^2)^{5/2}}$$

or; $$0.2094\,(r^2+z^2)^{5/2} = z^3$$

or; $$(r^2+z^2)^{5/2} = \frac{z^3}{0.209}$$

or; $$r^2+z^2 = \left(\frac{z^3}{0.209}\right)^{2/5}$$

or; $$r^2+z^2 = 1.87z^{1.2}$$

or; $$r = \sqrt{1.87z^{1.2}-z^2}$$

for $\sigma_v max$;

$$r = 0$$

or; $$0 = \sqrt{1.87z^{1.2}-z^2}$$

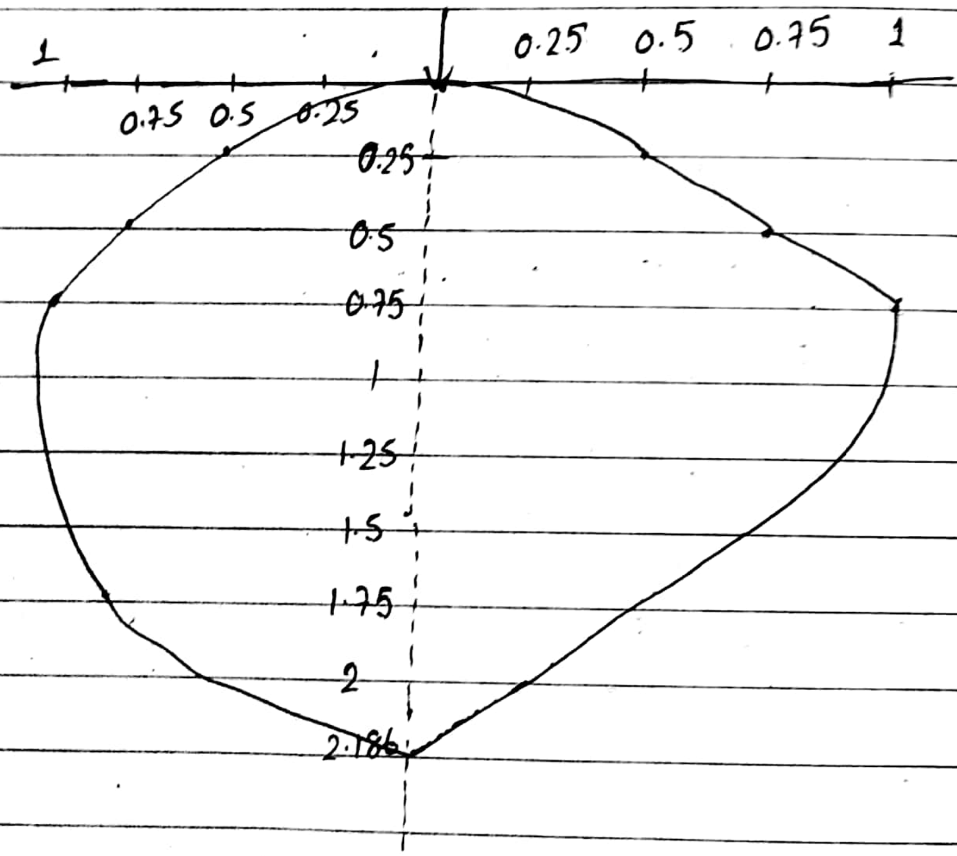
or, $1.87z^{1.2} = z^2$

or, $z = 2.186m$

for determining points

Assumed value of z

z (depth)	0.25	0.5	0.75	1.25	2.186
I_θ	0.00625	0.025	0.056	0.156	0.477
r/z	2.159	1.501	1.164	0.75	0
r	0.54	0.75	0.872	0.938	0



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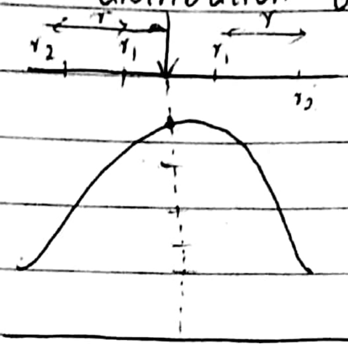
8.5 Vertical stress distribution diagrams

a) vertical stress distribution on a horizontal plane :-



r = vary
z = constant

Vertical stress distribution on a horizontal plane can be determined by using ~~Be~~ boussinesq eqⁿ;

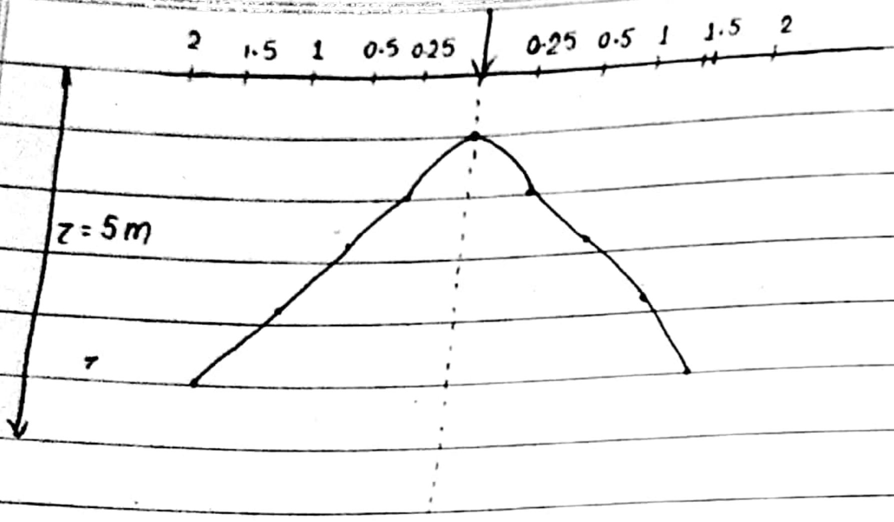
$$\sigma_v = \frac{Q}{z^2} \times I_B$$

$$I_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

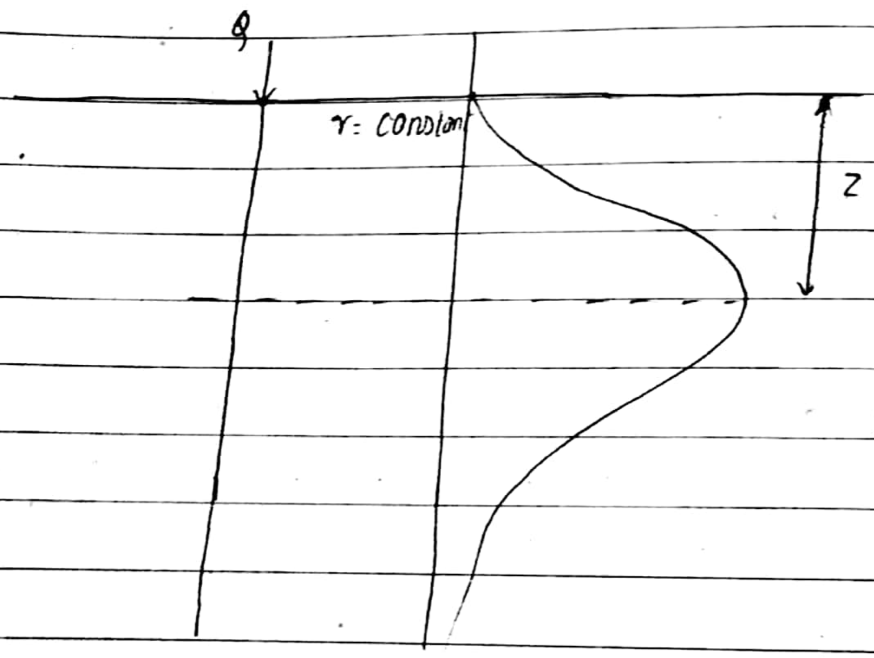
Q.1) A point load of 1000 kN acts on the ground surface. Show the variation of vertical stress on a horizontal plane at a depth of 5m below the ground surface.

r	0	0.25	0.5	1	1.5
r/z	0	0.05	0.1	0.2	0.3
I_B	0.477	0.472	~~0.376~~ 0.465	~~0.302~~ 0.432	~~0.247~~ 0.384
σ_v	19.08	~~16.88~~ 18.98	~~15.04~~ 18.6	~~12.08~~ 17.28	~~9.88~~ 15.36

$$\sigma_v = \frac{Q}{z^2} \times I_B. \qquad I_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$



b) Vertical stress distribution on a vertical plane :-



[r = constant]
z = vary.

$$\sigma_v = \frac{Q}{z^2} \times \frac{3}{2\pi} \times \left[\frac{1}{1+\left(\frac{r}{z}\right)^2} \right]^{5/2}$$

For max^m or min;

$$\frac{d\sigma_v}{dz} = 0$$

Or; $\frac{d}{dz}\left(\frac{Q}{z^2}\times\frac{3}{2\pi}\times\left[\frac{(z^2)^{5/2}}{(r^2+z^2)^{5/2}}\right]\right)=0$

Or; $\frac{d}{dz}\left[\frac{3Q}{2\pi z^2}\times\frac{z^5}{(r^2+z^2)^{5/2}}\right]=0$

Or; $\frac{d}{dz}\left[\frac{3Q}{2\pi}\times\frac{z^3}{(r^2+z^2)^{5/2}}\right]=0$

Or; $\frac{3Q}{2\pi}\left[\frac{(r^2+z^2)^{5/2}\cdot\frac{dz^3}{dz}-z^3\cdot\frac{d}{dz}(r^2+z^2)^{5/2}}{(r^2+z^2)^{5}}\right]=0$

Or; $\frac{3Q}{2\pi}\left[\frac{(r^2+z^2)^{5/2}\cdot 3z^2-z^3\cdot\frac{d(r^2+z^2)^{5/2}}{d(r^2+z^2)}\times\frac{d(r^2+z^2)}{dz}}{(r^2+z^2)^5}\right]=0$

Or; $\frac{3Q}{2\pi}\left[(r^2+z^2)^{5/2}\cdot 3z^2-z^3\cdot\frac{5}{\cancel{2}}(r^2+z^2)^{3/2}\times\cancel{2}z\right]=0$

Or; $\frac{3Q}{2\pi}\left[(r^2+z^2)^{5/2}\cdot 3z^2-5z^4(r^2+z^2)^{3/2}\right]=0$

Or; $(r^2+z^2)^{5/2}\cdot 3\cancel{z^2}=5z^4(r^2+z^2)^{3/2}$

Or; $\frac{(r^2+z^2)^{5/2}}{(r^2+z^2)^{3/2}}\,3=5z^2$

Or; $(r^2+z^2)3=5z^2$

Or; $\cancel{r^2+3z^2=5z^2}$

$\cancel{r^2=2z^2}$

Or; $\cancel{\frac{r}{z}=\sqrt{2}}$

or, $r^2+z^2 = \frac{5}{3} z^2$

or, $r^2 = \frac{2}{3} z^2$

or, $\frac{r}{z} = \sqrt{\frac{2}{3}}$

or, $\frac{r}{z} = 0.816$

or, $\boxed{\frac{r}{0.816} = z}$

Q.NO 2) A point load of 500 KN acts on a ground surface. Show the variation of vertical stress on a vertical plane at a radial distance 2m away from the applied load on the ground surface.

Solution;

$Q = 500\ KN$

r ~~z~~ $r = 2m$

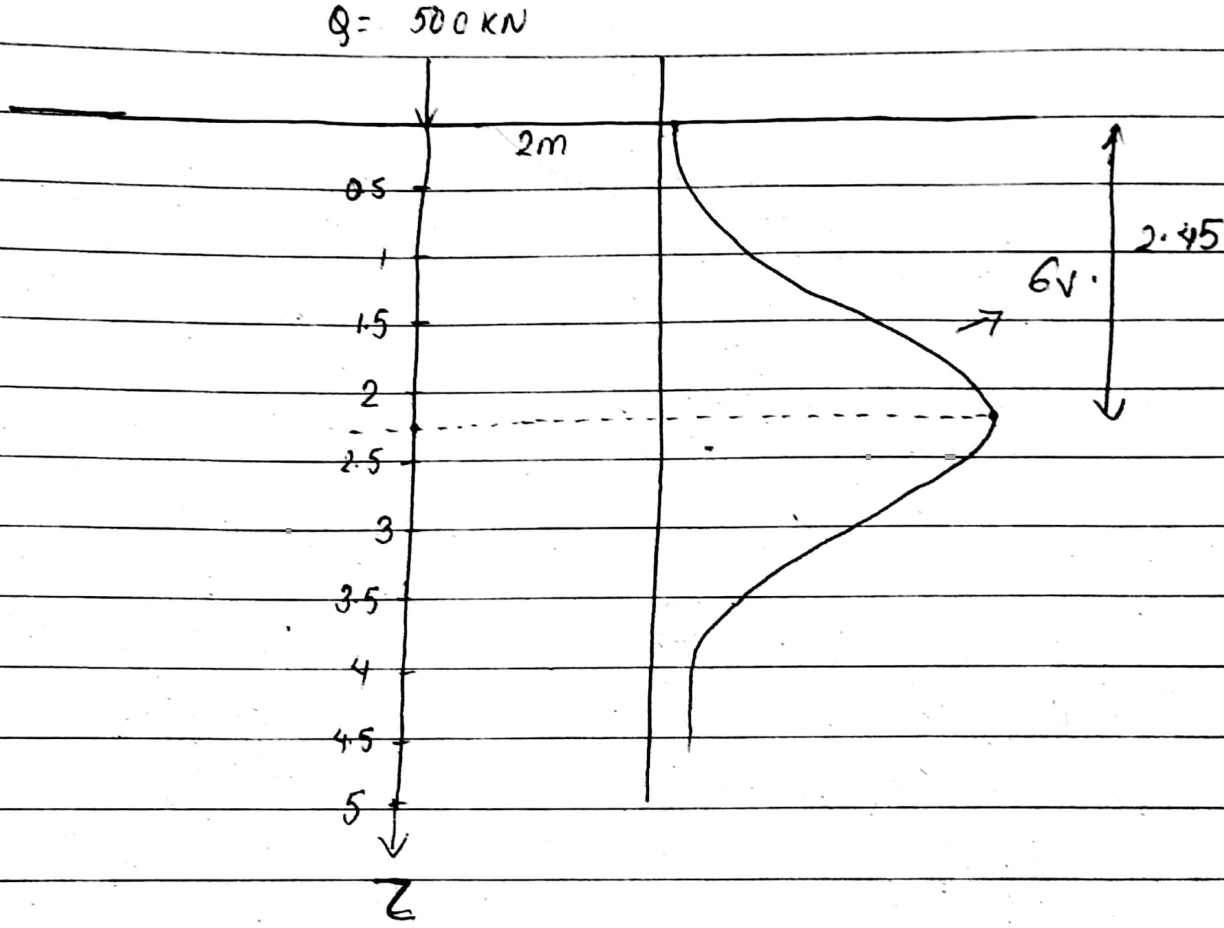
r/z for σ_v max,

$\frac{r}{z} = 0.816$

$\frac{2}{0.816} = z$

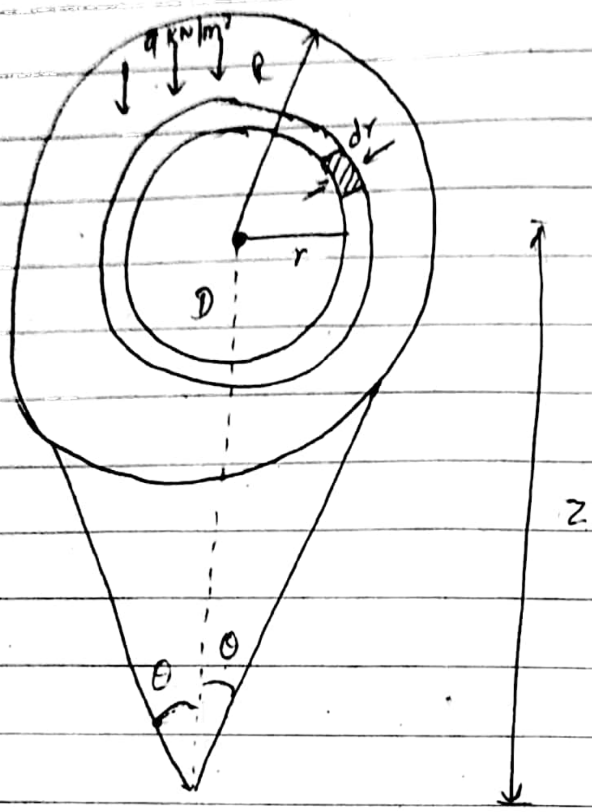
$z \simeq 2.45m$

z	1	2	2.45	2.5	3	4
r/z	2	1	0.816	0.8	0.67	0.5
I_B	0.0085	0.084	0.133	0.138	0.188	0.27
σ_v	4.25	10.5	11.07	11.04	10.44	8.43



8.6 Vertical stress distribution beneath the loaded area

a) Circular load :-



let us consider a circular area of radius R carrying a uniformly distributed load of intensity q kN/m^2. Consider an elementary ring of radius r and thickness dr of loaded area as shown. let θ be the apex angle at a depth z.

Using Boussinesq eqⁿ;

σ_v on a elementary ring be;

$$\sigma_v = \frac{Q}{z^2} \times \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

$$\Delta\sigma_v = \frac{q \times dA}{z^2} \times \frac{3}{2\pi} \times \left[\frac{z^5}{\left(r^2+z^2\right)^{5/2}}\right]$$

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or; $\Delta\sigma_v = \frac{q \times 2\pi r \cdot dr}{z^2} \times \frac{3}{2\pi} \times \left[\frac{z^5}{(r^2+z^2)^{5/2}}\right]$

$\Delta\sigma_v = \frac{3qr \cdot dr}{z^2} \times \frac{z^5}{(r^2+z^2)^{5/2}}$

for σ_v;
Integrating $\Delta\sigma_v$ from 0 to 'R'

$$\sigma_v = \int_0^R 3q \cdot z^3 \cdot \frac{r \cdot dr}{(r^2+z^2)^{5/2}}$$

$$= 3qz^3 \int_0^R \frac{r \cdot dr}{(r^2+z^2)^{5/2}}$$

let, $U = r^2 + z^2$

$\frac{du}{dr} = 2r$

$\frac{du}{2} = r \cdot dr$

when $r = 0$; $U = z^2$
$r = R$, $U = R^2 + z^2$

Now;

$$\sigma_v = 3qz^3 \int_{z^2}^{R^2+z^2} \frac{du}{2 \times U^{5/2}}$$

$$= \frac{3qz^3}{z} \int_{z^2}^{R^2+z^2} U^{-5/2} \cdot du$$

$$= \frac{3qz^3}{2}\left[\frac{u^{-5/2+1}}{-5/2+1}\right]_{z^2}^{R^2+z^2}$$

$$= \frac{\cancel{3}qz^3}{\cancel{2}} \times \frac{-\cancel{2}}{\cancel{3}}\left[(R^2+z^2)^{-3/2} - (z^2)^{-3/2}\right]$$

$$= -qz^3\left[\frac{1}{(R^2+z^2)^{3/2}} - \frac{1}{z^3}\right]$$

$$= -q\left[\frac{z^3}{(R^2+z^2)^{3/2}} - \frac{z^3}{z^3}\right]$$

$$= -q\left[\frac{(z^2)^{3/2}}{(R^2+z^2)^{3/2}} - 1\right]$$

$$= q\left[1 - \left(\frac{1}{\frac{R^2}{z^2}+\frac{z^2}{z^2}}\right)^{3/2}\right]$$

$$= q\left[1 - \left[\frac{1}{1+\left(\frac{R}{z}\right)^2}\right]^{3/2}\right]$$

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Q.NO.3 A ring footing of thickness ~~0.5m~~ 2m rest at a depth of 2m below the ground surface. It carries a load intensity of 150 KN/m^2. The vertical stress at a depth of 2m along the axis of footing base be 39.64 KN/m^2. calculate the internal & external diameter of the footing.

Given;

$$t = \frac{D_e - D_i}{2}$$

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2019 Fall

Vertical stress due to rectangular load;

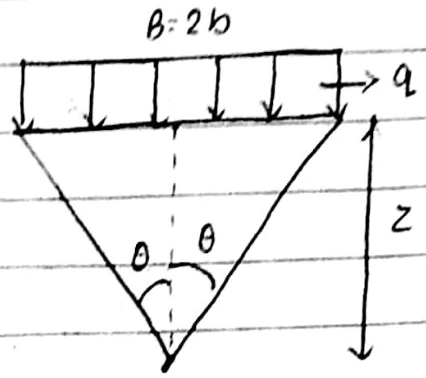
$$\sigma_v = q\left(I_{N_1} + I_{N_2} + I_{N_3} + \dots\right)$$

where;

In radian

$$I_N = \frac{1}{4\pi}\left[\left(\frac{2mn \times \sqrt{m^2+n^2+1}}{m^2+n^2+1+m^2n^2}\right)\left(\frac{m^2+n^2+2}{m^2+n^2+1}\right) + \boxed{\tan^{-1}\left(\frac{2mn \times \sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right)}\right]$$

② vertical stress due to strip load:-



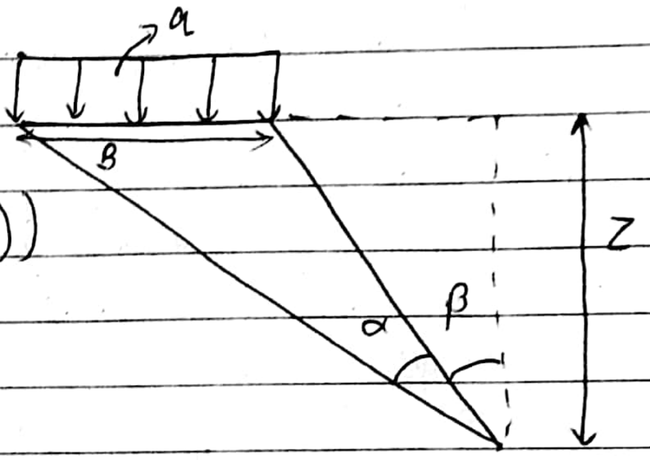
a) At centre

$$\sigma_v = \frac{q}{\pi}\left(2\theta + \sin 2\theta\right)$$

$$\theta = \tan^{-1}\left(\frac{b}{z}\right)$$

θ in radian

b) below the edge:-

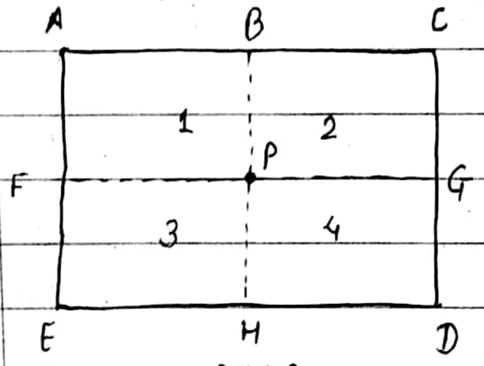


$$\sigma_v = \frac{q}{\pi}\left(\alpha + \sin\alpha\cos\left(\alpha + 2\beta\right)\right)$$

8.7 Newmark's and fadum's chart.

fadum chart

a) Point anywhere beneath the rectangular area



$$\sigma_v = q\left(IN_1 + IN_2 + IN_3 + IN_4\right)$$

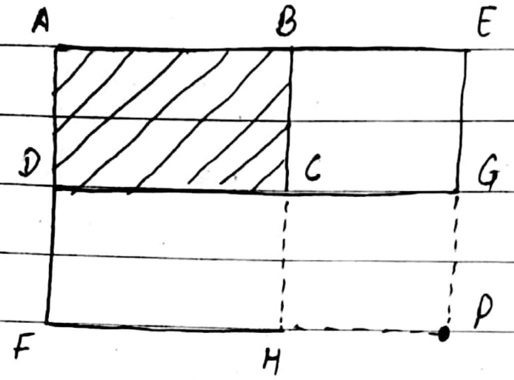
IN_1 ↓ ABPF

IN_2 ↑ BCGP

IN_3 ↓ HPFE

IN_4 ↑ HPGD

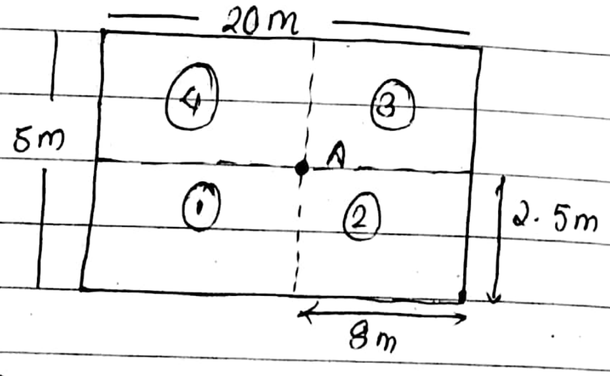
b) Point outside the loaded area:-



$$\sigma_v = q\left(I_{N_{AEPF}} - I_{N_{BEPH}} - I_{N_{DGPF}} + I_{N_{CGPH}}\right)$$

2017 fall

24) A strip footing is given in plan as shown in figure. The total load per unit area is $300\,kN/m^2$. Determine the intensity of vertical stress at a point 5m directly below the point A.



For; 1 & 4

$l = 12$

$B = 2.5$

$$m = \frac{B}{z} = \frac{2.5}{5} = 0.5$$

$$n = \frac{l}{z} = \frac{12}{5} = 2.4$$

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from fadum chart;

$IN_1 = 0.12$

for area 3, 2

$l = 8$

$B = 2.5$

$m = \frac{B}{z} = \frac{2.5}{5} = 0.5$

$n = \frac{l}{z} = \frac{8}{5} = 1.6$

from fadum chart

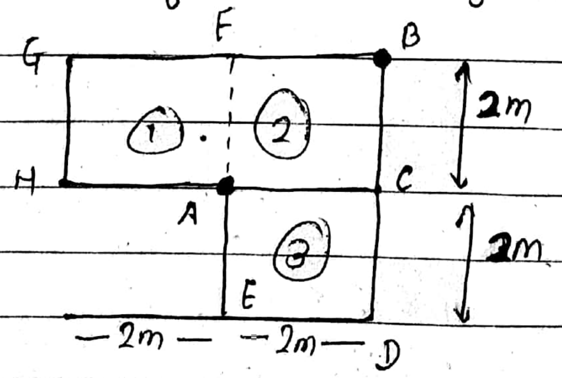
$IN = 0.11$

$\sigma_v = q(IN_1 + IN_2 + IN_3 + IN_4)$

$= 300(0.12 + 0.11 + 0.11 + 0.12)$

$= 138 \text{ kN/m}^2$

12 Using fadum's chart, determine the increase in vertical stress at 2m depth below A & B due to UDL of 40 kN/m^2 on the ground surface shown by area in fig.



for area 1,2,3

$l = 2m$

$B = 2m$

$m = \frac{b}{z} = \frac{2}{2} = 1$

$n = \frac{2}{2} = 1.$

from fadum chart;

$IN = 0.163$

$\sigma_v = q\,(IN_1 + IN_2 + IN_3)$

$= 40(0.163 \times 3)$

$= \cancel{13.04\ kN/m^2}$

$= 19.56\ kN/m^2.$

At point B

for area GBCH

$l = 4m$

$b = 2m$

$m = 1$

$n = 2$

$IN_1 = 0.2$

for area BFDE

$l = 4m$ $\quad$ $m = 1$

$b = 2m$ $\quad$ $n = 2$

$IN_2 = 0.2$

For area ACBF

$l = 2m$

$b = 2m.$

$m = 1$

$n = 1$

$IN_3 = 0.163.$

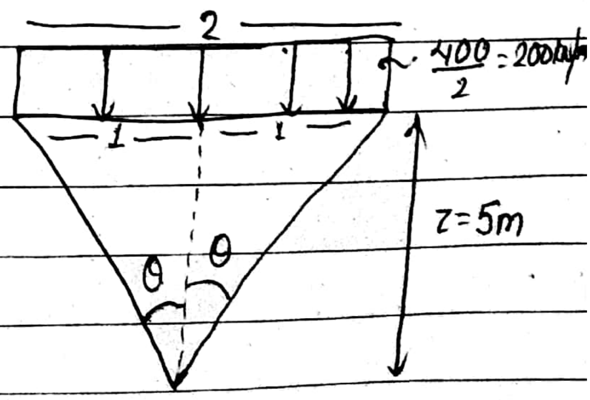
Now;

$$\sigma_v = q(IN_1 + IN_2 - IN_3)$$
$$= 40(0.2 + 0.2 - 0.163)$$
$$= 9.48 \text{ kN/m}^2.$$

q → small

18 A strip footing of width 2m carries a load of 400 kN/m. Calculate the maximum stress at a depth of 5m below the centre line of footing. Compare the result with 2:1 distibution method.

$$q = \frac{400}{2} = 200 \text{ kN/m}^2$$



Now;

From figure;

$$\tan\theta = \frac{b}{2}$$

$$\theta = \tan^{-1}\left(\frac{1}{5}\right) \quad [\theta \text{ in radian}]$$

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$= 0.197$

we have

$$\sigma_v = \frac{q}{\pi}\left(2\theta + \sin 2\theta\right)$$

$$= \frac{200}{\pi}\left[2\times 0.197 + \sin 2\times 0.197\right)$$

$$= 49.61 \ kN/m^2$$

from 2:1 method;

$$\sigma_v = \frac{q\times B}{B+z}$$

$$= \frac{200\times 2}{2+5}$$

$$= 57.14 \ kN/m^2$$

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Newmark's chart:-

This chart is based on the concept of vertical stress below the centre of circular area. A loaded circular area is divided into 20 sectors. If q is the intensity of loading the vertical stress σ_v at a depth z below the centre is given by;

$$\sigma_v = q\left[1 - \left(\frac{1}{1+\left(\frac{r}{z}\right)^2}\right)^{3/2}\right]$$

Assumption of Newmark's chart

1) Influence of each area unit in carrying a load stresses is equal.
2) Stresses below the newmark's chart is considered to be negligible.

For standard Newmark's chart number of concentric circle (m) = 10
number of radial line (n) = 20

No. of area units = $m \times n$
= 10×20
= 200

Procedure to find the vertical stress using Newmark's chart:-

Place the point at which the stresses are to be calculated at the centre of the chart and count the number of area units overlapped. Then vertical stress (σ_v) is given by:

$$\sigma_v = I_f n q$$

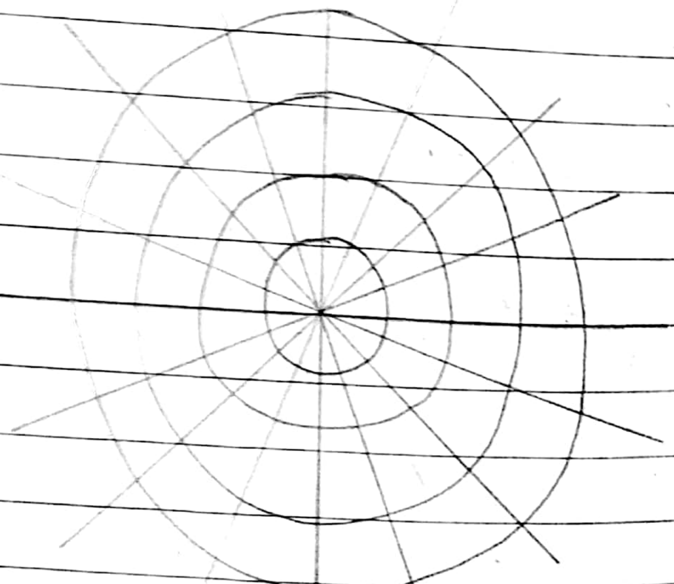
Where;

I_f = Influence factor
= 0.005 (standard value)

$$I_f = \frac{1}{m \times n}$$

n = no. of radial units

Use of Newmark's chart
To determine the vertical stress due to the loaded area of any shape, irregular geometry at any point below the loaded area.



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