

A
Complete Tutorial Solution
on

Fluid Mechanics

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Tutorial 1

Fluid and its Physical Properties

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1. If 6 m^3 of oil weighs 47 kN , find its specific weight, density and relative density
Solution;

Given;

$$\text{Volume of oil (V)} = 6\text{ m}^3$$

$$\text{Weight of oil (W)} = 47\text{ kN}$$

$$\text{Specific weight, } \gamma = \frac{W}{V} = \frac{47\text{ kN}}{6\text{ m}^3} = 7.83\text{ kN/m}^3.$$

$$\text{Density } (\rho) = \frac{m}{V} = \frac{W}{g} = \frac{47\text{ kN}}{9.81 \times 6} = \frac{47 \times 10^3}{58.86} = 798.50\text{ kg/m}^3.$$

$$\text{Relative density (S)} = \frac{\gamma_i}{\gamma_{H_2O}} = \frac{7.83 \times 10^3}{9810} = 0.798.$$

2. At a certain depth in the ocean, the pressure is 80 MPa . Assume that the specific weight at the surface of the earth is 10 kN/m^3 and the average bulk modulus is 2340 GPa . Find:

- the change in specific volume between the surface and the large depth.
- the specific volume at the depth and
- the specific weight at the depth.

Solution;

Given;

$$\text{Pressure at certain depth (P)} = 80 \times 10^3\text{ Pa.}$$

$$\text{Specific weight at surface } (\gamma) = 10 \times 10^3\text{ N/m}^3.$$

$$\text{Average Bulk Modulus (K)} = 2340 \times 10^6\text{ Pa.}$$

We have;

$$\gamma = \rho g.$$

$$\rho = \frac{\gamma}{g} = \frac{10,000}{9.81} = 1019.4\text{ N/m}^2.$$

$$K = -\frac{dp}{\frac{dV}{V}} \quad \text{or, } dV = \frac{80 \times 10^3}{2340 \times 10^9} \times 1019.4.$$

$$\therefore dV = -0.335 \times 10^{-4}\text{ m}^3/\text{kg}.$$

$$V_{\text{depth}} = V + \Delta V = 9.475 \times 10^{-4}\text{ m}^3/\text{kg}.$$

$$\gamma = \rho g = 1019.4 \times 9.81 = 10.35\text{ kN/m}^3.$$

Albhar

3. Eight Kilometers below the surface of the ocean the pressure is 81.7 MPa . Calculate the density of sea water at this depth, if the density at the surface is 1025 kg/m^3 and average bulk modulus of elasticity is 2.344 GPa . Assume unit mass.

Solution:

Pressure below the ~~ocean~~ at $8 \text{ km} = 81.7 \times 10^6 \text{ Pa}$.

Bulk modulus of elasticity (K) $= 2.344 \text{ GPa}$.

Density of sea water at surface (ρ_1) $= 1025 \text{ kg/m}^3$.

Let ρ_2 be the density at given depth. If ΔV be the change in volume then;

$$\Delta V = \frac{1}{\rho_1} - \frac{1}{\rho_2}$$

$$= \frac{\rho_2 - \rho_1}{\rho_1 \cdot \rho_2}$$

$$\Delta P = K \cdot \frac{\Delta V}{V}$$

$$\therefore 81.7 \times 10^6 = 2.344 \times 10^9 \times \frac{\rho_2 - 1025}{\rho_2 \cdot 1025 \times \frac{1}{1025}}$$

$$\therefore 27.7 \times 10^3 = 1025 \times 28.7$$

$$\therefore \rho_2 = 1062.01 \text{ kg/m}^3$$

Hence, the required density is 1062.01 kg/m^3 .

4. Calculate the specific weight, density and specific gravity of one litre of liquid which weighs 7 N .

Solution:

$$\text{Given, Volume of liquid (V)} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3.$$

$$\text{Weight of liquid (W)} = 7 \text{ N}.$$

$$\text{Specific weight (}\gamma\text{)} = \frac{W}{V} = \frac{7}{\frac{1}{1000}} = 7000 \text{ N/m}^3.$$

$$\text{Density (}\rho\text{)} = \frac{m}{V} = \frac{W}{gV} = \frac{7}{9.81 \times \frac{1}{1000}} = \frac{7000}{9.81} = 713.55 \text{ kg/m}^3.$$

$$\text{Specific gravity (S)} = \frac{\rho_l}{\rho_{H_2O}} = \frac{713.55}{1000} = 0.7135.$$

Albhar

5. The viscosity of pure water at 0°C is 0.01793 poise and density is $1\text{g}/\text{cm}^3$. Express dynamic viscosity and the kinematic viscosity in SI units.

Solution:

Given, Viscosity of water at 0°C (η) = 0.01793 poise = $0.01793 \times 0.1 \text{ Pa}\cdot\text{s}$
 $= 0.001793 \text{ Pa}\cdot\text{s}$

Density of water (ρ) = $1000 \text{ kg}/\text{m}^3$.

Dynamic viscosity = $0.001793 \text{ Pa}\cdot\text{s}$
 $= 1.793 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2$.

$\therefore$ Kinematic viscosity = $\frac{\eta}{\rho} = \frac{1.793 \times 10^{-3}}{10^3} = 1.793 \times 10^{-6} \text{ N}\cdot\text{s}/\text{m}^2$.

6. Express the kinematic viscosity in Stokes for a liquid with specific gravity 0.95 and dynamic viscosity 0.011 poise.

Solution:

Given; dynamic viscosity (μ) = 0.011 poise = $0.0011 \text{ Pa}\cdot\text{s}$.

Specific gravity (S) = 0.95.

or, $S = \frac{\gamma_l}{\gamma_{H_2O}}$ or, $0.95 = \frac{\gamma_l}{\gamma_{H_2O}}$ or, $0.95 = \frac{\gamma_l}{9810}$ $\therefore \gamma_l = 9319.50$.

we have; $\gamma_l = \rho \times g$.

$\therefore \rho = \frac{9319.50}{9.81} = 950 \text{ kg}/\text{m}^3$.

Now; $\nu = \frac{\mu}{\rho} = \frac{0.0011}{950} = 1.1578 \times 10^{-6}$.

$\therefore \nu = \frac{1.1578 \times 10^{-6}}{10^{-4}} \text{ Stokes} = 0.011578 \text{ Stokes}$.

7. Determine the bulk modulus of elasticity of a liquid. If the pressure of the liquid increased from $70 \text{ N}/\text{cm}^2$ to $130 \text{ N}/\text{cm}^2$. The volume of the liquid decreases by 0.15 percent.

Solution:

Given, change in pressure (Δp) = $60 \text{ N}/\text{cm}^2$.

Decrease in volume = 0.15%.

Now; Bulk modulus (K) = $-\frac{V \Delta p}{\Delta V} = -\frac{V \times 60}{\frac{0.15}{100} \times V}$.

$= 40000 \text{ N}/\text{cm}^2$.

$\therefore K = 4 \times 10^4 \text{ N}/\text{cm}^2$.

311bhar

8. The volume of a liquid is reduced by 1% by increasing the pressure from 5 atmospheres to 125 atmospheres. Estimate the modulus of elasticity of the liquid.

Solution;

we have;

$$\text{Change in Pressure } (\Delta P) = 125 \text{ atm} - 5 \text{ atm} = 120 \text{ atm}$$

$$= 120 \times 101325 \text{ Pa.}$$

$$= 1.2159 \times 10^7 \text{ Pa.}$$

$$\text{Change in volume in \% } \left(\frac{\Delta V}{V} \right) = -1\%$$

Now;

$$\text{Modulus of elasticity } (K) = - \frac{\Delta P}{\frac{\Delta V}{V}}$$

$$\text{or } K = \frac{1.2159 \times 10^7}{\frac{-1}{100}}$$

$$\text{or } K = 1.2159 \times 10^9 \text{ N/m}^2$$

$$\therefore K = 1.216 \times 10^6 \text{ KN/m}^2.$$

$\therefore$ The required modulus of elasticity is $1.216 \times 10^6 \text{ KN/m}^2$.

9. A soap bubble 0.8 cm in diameter has a gauge pressure of 1 KN/m^2 within it. Estimate the surface tension of the soap solution.

Solution;

Let ' σ ' be the surface tension of soap solution

Given;

$$\text{Diameter of soap bubble } (d) = 0.8 \text{ cm} = 8 \times 10^{-3} \text{ m.}$$

$$\therefore r = 4 \times 10^{-3} \text{ m.}$$

$$\therefore \sigma = \frac{Pr}{2 \times 2}$$

$$= \frac{1 \times 4 \times 10^{-3} \times 10^3}{4}$$

$$= 1 \text{ N/m.}$$

311bhan

10. Determine the height that 20°C water would climb in a vertical 0.02cm diameter tube if it attaches to the wall with an angle β of 30° to tube vertical, take $\sigma = 0.0737\text{N/m}$.

Solution:

Given, Surface tension (σ) = 0.0737N/m^2 .

Diameter of tube (d) = $0.02\text{cm} = 0.02 \times 10^{-2}\text{m}$.

Angle of contact (β) = 30° .

Specific weight of water ($\gamma_{\text{H}_2\text{O}}$) = 9810N/m^3 .

Now,

$$\text{Height } (h) = \frac{4\sigma \cos\beta}{\gamma d}$$

$$\text{or, } h = \frac{4 \times 0.0737 \times \cos 30^{\circ}}{9810 \times 0.02 \times 10^{-2}}$$

$$\therefore h = 0.13\text{m}$$

$\therefore$ The height that 20°C water would climb is 0.13m in a 0.02cm diameter tube.

11. Mercury makes an angle of 130° when in contact with clean glass. What distance will mercury depress in a vertical 2mm diameter glass tube, take $\sigma = 0.5\text{N/m}$.

Solution:

Let ' h ' be the height of capillary fall.

Angle of contact (β) = 130° .

We have, $\sigma = 0.5\text{N/m}$.

Radius = $1\text{mm} = 1 \times 10^{-3}\text{m}$.

Now,

$$h = \frac{2\sigma \cos\beta}{\rho \times g \times r}$$

$$= \frac{2 \times 0.5 \times \cos 130^{\circ}}{13600 \times 9.81 \times 10^{-3}}$$

$$[\because \rho_{\text{mercury}} = 13600\text{kg/m}^3]$$

$$= -0.48\text{cm}$$

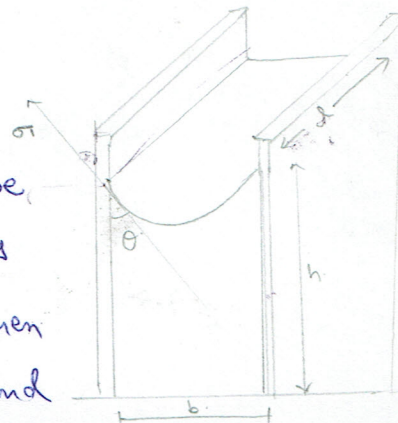
$\therefore$ The height raised is -4.8mm .

Alibhax

12. When two plates are placed vertically at a distance 'b' apart, derive the equation showing increased height of the liquid surface between the plates due to capillary. Also when flat plates of glass are used with a 1mm gap, what is the increased height of the water surface?

Solution;

Two plates are kept 'b' distance apart. If β be the angle of contact between liquid and glass plate and σ be the surface tension of liquid then it can be resolved into two component vertical and horizontal.



Horizontal component ($\sigma \cos \beta$) being equal and opposite cancel each other while wt. of liquid is balanced by vertical component.

Vertical component ($\sigma \sin \beta$) = $\sigma \cos \beta$.

Total vertical component = $\sigma \cos \beta \times 2l$.

Now;

In equilibrium condition,

Upward force = Downward force.

$$\sigma \cos \beta \times 2l = mg = \rho V g = \rho g \cdot A h.$$

$$\sigma \cos \beta \times 2l \times b \times \rho \times g = 2 \sigma \cos \beta \times l.$$

$$\sigma \cos \beta \times h = \frac{2 \sigma \cos \beta}{\rho \times g \times b}.$$

$$\therefore h = \frac{2 \sigma \cos \beta}{\rho g b}.$$

Then, if $b = 10^{-3} \text{ m}$,

$$\beta = 0^\circ.$$

$$\sigma = 0.0737 \text{ N/m}$$

$$\rho = 1000 \times 9.81 = 9810 \text{ N/m}^3$$

$$\Rightarrow h = \frac{2 \times 0.0737 \times 1}{9810 \times 10^{-3}}$$

$$\therefore h = 0.015 \text{ m}.$$

$$\therefore = 1.5 \text{ cm}.$$

Albhar

13. The surface tension of water in contact with air at 20°C is 0.0725 kgf/m . The pressure inside a droplet of water is to be 0.0020 kgf/cm^2 . Calculate the diameter of the droplet of water.

solution;

Given;

Surface tension of water at 20°C (σ) = 0.0725 kgf/m .

Pressure (P) = 0.0020 kgf/cm^2

$$= 0.0020 \times 10000 \text{ m}^2.$$

$$= 20 \text{ m}^2.$$

we have;

$$\sigma = \frac{\Delta P \cdot d}{4}$$

$$\text{or, } 0.0725 = \frac{20 \times d}{4}$$

$$\text{or, } d = \frac{4 \times 0.0725}{20}$$

$$\therefore d = 0.0145 \text{ m}$$

$\therefore$ The diameter of the tube is found to be 1.45 cm .

14. Water at 10°C ($\sigma = 0.0742 \text{ N/m}$ and $\gamma = 9804 \text{ N/m}^2$) stands in a clean glass tube of 2 mm diameter at a height of 35 mm . What is the true static height?

solution;

Given;

Surface tension (σ) = 0.0742 N/m

$\gamma = 9804 \text{ N/m}^2$

diameter (d) = 2 mm

$$= 2 \times 10^{-3} \text{ m}$$

$\therefore$ radius (r) = $1 \times 10^{-3} \text{ m}$

height (h) = $35 \text{ mm} = 35 \times 10^{-3} \text{ m}$

Now;

$$h = \frac{2\sigma}{\gamma R}$$

$$= \frac{2 \times 0.0742}{9804 \times 10^{-3}}$$

$$= 0.015 \text{ m}$$

$$\therefore \text{Static height} = 35 \times 10^{-3} \text{ m} - 15 \times 10^{-3} \text{ m}$$

$$= 20 \times 10^{-3} \text{ m}$$

311bhaan

15. Determine the density and the specific gravity of air at standard conditions (i.e. 15°C and 101.3 kPa absolute).

Solution;

Given;

$$\text{Temperature } (T) = 15^{\circ}\text{C} = 273.15 + 15 = 288.15\text{ K}$$

$$\text{Pressure } (P) = 101.3\text{ kPa} = 101.3 \times 10^3\text{ Pa}$$

$$\text{we have, } R \text{ for air} = 286.8\text{ (J/kg}\cdot\text{K)}$$

Now;

$$PV = mRT$$

$$\text{or, } P = \frac{m}{V} \cdot R \cdot T$$

$$\text{or, } P = \rho \cdot R \cdot T$$

$$\text{or, } 101.3 \times 10^3 = \rho \times 286.8 \times 288.15$$

$$\text{or, } \rho = \frac{101.3 \times 10^3}{286.8 \times 288.15}$$

$$\therefore \rho = 1.226\text{ kg/m}^3$$

Now;

$$\gamma = \rho \times g$$

$$= 1.226 \times 9.81$$

$$= 12.02\text{ N/m}^3$$

$\therefore$ The density and specific weights are found to be 1.226 kg/m^3 and 12.02 N/m^3

Albhar

16. A 5 m^3 air tank is pressurized to 5 MPa absolute. When the temperature reaches 8°C , calculate the density and the air mass.

Solution;

Given;

$$\text{Volume of air (V)} = 5\text{ m}^3$$

$$\text{Pressure (P)} = 5\text{ MPa} = 5 \times 10^6\text{ Pa}$$

$$\text{Temperature (T)} = 281\text{ K}$$

we have;

$$P = \rho R T$$

$$\text{or, } \rho = \frac{P}{RT}$$

$$\text{or, } \rho = \frac{5 \times 10^6}{286.7 \times 281}$$

$$\therefore \rho = 62.06\text{ kg/m}^3$$

$$\text{And; } \rho = \frac{m}{V}$$

$$\text{or, } m = \rho \times V$$

$$\text{or, } m = 62.06 \times 5$$

$$\therefore m = 310.31\text{ kg}$$

$\therefore$ The density is 62.06 kg/m^3 and mass is 310.31 kg .

17. A 5 m^3 air tank is evacuated until the pressure is 20 Pa absolute. Find the density and air mass if the temperature is 18°C .

Solution;

Given;

$$\text{Volume (V)} = 5\text{ m}^3$$

$$\text{Pressure (P)} = 20\text{ Pa}$$

$$\text{Temperature (T)} = 273 + 18 = 291\text{ K}$$

we have;

$$PV = mRT$$

$$\text{or, } P = \rho R T$$

$$\text{or, } P = 286.8 \times 291 \times \rho$$

$$\text{or, } 20 = 286.8 \times 291 \times \rho$$

$$\therefore \rho = 2.394 \times 10^{-4}\text{ kg/m}^3$$

$$\text{now, } \rho = \frac{m}{V} \text{ or, } 2.394 \times 10^{-4} = \frac{m}{5}$$

$$\therefore m = 1.19 \times 10^{-3}\text{ kg}$$

Alibhara

18. For $K = 2.2 \text{ GPa}$ on bulk modulus of elasticity of water, what pressure is required to reduce its volume by 0.5%?

Solution;

Given; Bulk modulus of elasticity, $(K) = 2.2 \times 10^9 \text{ Pa}$.

Volume decreased by 0.5%.

Now;

$$K = \frac{-P}{\frac{\Delta V}{V}}$$

$$\text{or, } 2.2 \times 10^9 = \frac{-P}{\frac{-0.5}{100}}$$

$$\text{or, } P = 110000000 \text{ Pa}$$

$$\therefore P = 11 \text{ MPa}$$

$\therefore$ The required pressure is 11 MPa.

19. A U-tube is made up of two capillaries of diameter 10mm and 15mm respectively. The U-tube is kept vertically and partially filled with water of surface tension 0.0075 kgf/m and zero contact angles. Calculate the difference in the level of the menisci caused by the capillary.

Solution;

Surface tension $(\sigma) = 0.0075 \text{ kgf/m}$

Diameter $(d_1) = 10^{-3} \text{ m}$

Diameter of 2nd tube $(d_2) = 1.5 \times 10^{-3} \text{ m}$

Now;

$$\begin{aligned} \text{height rise in 1st tube } (h_1) &= \frac{4\sigma}{\rho \times g \times d_1} \\ &= \frac{4 \times 0.0075}{1000 \times 9.81 \times 10^{-3}} \\ &= 0.03 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{In 2nd tube } (h_2) &= \frac{4 \times 0.0075 \times 9.81 \times 2}{1000 \times 9.81 \times 10^{-3} \times 1.5} \\ &= 0.02 \text{ m} \end{aligned}$$

$$\therefore h_2 - h_1 = 1 \text{ cm}$$

3116haz

20. A 100mm deep stream of water is flowing over a boundary. It is considered to have zero velocity at the boundary and 1.5 m/s at the free surface. Assuming a linear velocity profile, what is the shear stress in the water?

Solution;

Given;

$$\text{Height (dy)} = 100 \text{ mm}$$

$$\text{velocity (dv)} = 1.5 \text{ m/s}$$

$$\mu \text{ for water} = 10^{-3} \text{ Pa}\cdot\text{s}$$

Now;

$$\frac{du}{dy} = \frac{1.5}{100 \times 10^{-3}} = 15$$

$$\text{Shear stress, } \tau = \mu \frac{du}{dy} = 15 \times 10^{-3} \text{ Pa}$$

$$\therefore \tau = 15 \times 10^{-3} \text{ Pa}$$

$\therefore$ The shear stress is $15 \times 10^{-3} \text{ Pa}$.

21. A velocity distribution of a 2cm diameter pipe is measured to be $V(r) = 10(1 - r^2/r_0^2)$ m/s. where r_0 is the radius of the pipe. Calculate the shear stress at the wall if the water at 25°C , with viscosity $0.9 \times 10^{-3} \text{ Pa}\cdot\text{s}$ is flowing.

Solution;

Given;

$$V(r) = 10(1 - r^2/r_0^2) \text{ m/s}$$

$$\text{diameter (d)} = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

we have;

$$\begin{aligned} \frac{dV(r)}{dr} &= \frac{d}{dr} \left(10 - \frac{10}{r_0^2} r^2 \right) \\ &= -\frac{20r}{r_0^2} \end{aligned}$$

$$\mu = 0.9 \times 10^{-3} \text{ Pa}\cdot\text{s}$$

$$\text{Shear stress; } \tau = \mu \frac{dV}{dr} = 0.9 \times 10^{-3} \left(-\frac{20r}{r_0^2} \right)$$

At the wall, $r = r_0$.

$$\begin{aligned} \therefore \tau &= 0.9 \times 10^{-3} \times \left(-\frac{20}{0.01} \right) \\ &= -1.8 \text{ Pa} \end{aligned}$$

Albhar

22. For two 0.2m long rotating concentric cylinders the velocity distribution is given by $v(r) = 0.4/r - 1000r$ m/s. If the diameters of the cylinders are 2cm and 4cm respectively, calculate the fluid viscosity, if the torque on the inner cylinder is measured to be 0.0026 N·m.

Solution;

Given; $v(r) = \frac{0.4}{r} - 1000r$ m/s.

$d_1 = 2\text{cm} = 0.02\text{m}$

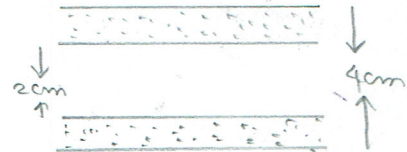
$d_2 = 4\text{cm}$

$r_1 = 0.01\text{m}$

$d_2 = 0.04\text{m}$

$l = 0.2\text{m}$

$r_2 = 0.02\text{m}$



Now;

$$\frac{dv}{dr} = \frac{-0.4}{r^2} - 1000$$

At $r = 0.01\text{m}$, $\frac{dv}{dr} = -5000$.

Given, Torque $= S \cdot F \times r_1$.

$$\therefore S \cdot F = \frac{0.0026}{0.01} = 0.26\text{N}$$

Now, Shear stress in inner cylinder $= \frac{0.26}{2\pi r_1 l} = 20.7\text{Pa}$.

$$\tau = \mu \cdot \frac{du}{dr}$$

$$\text{or, } \mu = \frac{\tau}{\frac{du}{dr}}$$

$$= \frac{20.7}{5000}$$

$$= 4.14 \times 10^{-3} \text{Ns/m}^2$$

$\therefore$ The required fluid viscosity is $4.14 \times 10^{-3} \text{Ns/m}^2$.

311bhard

23. A flat plate $250\text{mm} \times 800\text{mm}$ slides on oil with viscosity $0.65 \text{ N}\cdot\text{s}/\text{m}^2$ over a large plate surface. What force is required to drag the plate at 1.5 m/s , if the separating oil film is 0.5 mm thick.

Solution;

Given;

$$\text{thickness } (t) = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$$

$$\text{Area } (A) = 250 \text{ mm} \times 800 \text{ mm} = 0.2 \text{ m}^2$$

$$\mu = 0.65 \text{ N}\cdot\text{s}/\text{m}^2$$

$$\text{velocity } (u) = 1.5 \text{ m/s}$$

$$\text{Now; } \frac{du}{dy} = \frac{u}{t} = 3000$$

$$\begin{aligned} \text{Shear stress; } T &= \mu \cdot \frac{du}{dy} \\ &= 0.65 \times 3000 \\ &= 1950 \text{ Pa} \end{aligned}$$

$$\text{Force} = T \times A = 1950 \times 0.2 = 390 \text{ N}$$

$\therefore$ The shear stress is 1950 Pa and force is 390 N .

24. Calculate the dynamic viscosity of oil, which is used for lubrication between a square plane of size $0.8 \text{ m} \times 0.8 \text{ m}$ and an inclined plane with angle of inclined 30° . The weight of the plates is 30 kgf and it slides down the inclined plane with a uniform velocity of 0.3 m/sec . Take the thickness of oil film is 1.5 mm .

Solution;

Given;

$$\text{Area } (A) = 0.8 \text{ m} \times 0.8 \text{ m} = 0.64 \text{ m}^2$$

$$\text{velocity } (v) = 0.3 \text{ m/s}$$

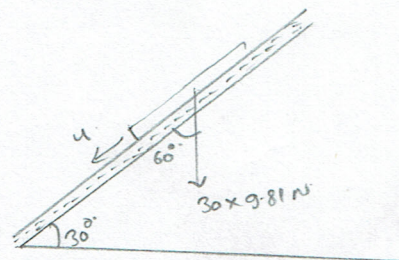
$$\text{thickness } (t) = 1.5 \times 10^{-3} \text{ m}$$

$$\begin{aligned} \text{Shear force} &= 30 \times 9.81 \times \sin 30^\circ \\ &= 147.15 \text{ N} \end{aligned}$$

$$T = \frac{F}{A} = \frac{147.15}{0.64} = 229.92 \text{ Pa}$$

$$\frac{du}{dy} = \frac{u}{t} = \frac{0.3}{1.5 \times 10^{-3}} = 200$$

$$\mu = \frac{T}{\frac{du}{dy}} = \frac{229.92}{200} \therefore \mu = 0.117 \text{ kg}\cdot\text{s}/\text{m}^2$$



311bhar

25. A 1m^2 , 10kg float plate slides down a 30° inclined plane over a 0.1cm uniform layer of oil with viscosity 0.05Ns/m^2 . Determine the steady state velocity.

Solution;

Given, Area (A) = 1m^2 .

thickness (t) = $0.1\text{cm} = 1 \times 10^{-3}\text{m}$.

$\mu = 0.05\text{Ns/m}^2$.

mass (m) = 10kg .

Now;

Force (F) = $mg \sin \theta = 10 \times 9.81 \times \sin 30^\circ = 49.05\text{N}$.

Shear stress (τ) = $\frac{F}{A} = 49.05\text{Pa}$.

$\frac{du}{dy} = \frac{\tau}{\mu} = 1000u$.

we have; $\tau = \mu \cdot \frac{du}{dy}$.

$\therefore 49.05 = 0.05 \times 1000u$.

$\therefore u = 0.98\text{m/s}$.

$\therefore$ The required velocity is 0.98m/s .

26. A long pipe of diameter 125mm is placed inside a concentric cylinder of diameter 122mm and length 200m . An oil film is introduced in the gap between the pipe and the cylinder. What force is necessary to move the cylinder at a velocity of 1m/s . Assume that the kinematic viscosity of oil is 30cst and the specific gravity is 0.9 .

Solution;

Kinematic viscosity (ν) = $30\text{cst} = 30 \times 10^{-2}\text{st}$.
 $= 30 \times 10^{-2} \times 10^{-4}\text{m}^2/\text{s}$.
 $= 3 \times 10^{-5}\text{m}^2/\text{s}$.

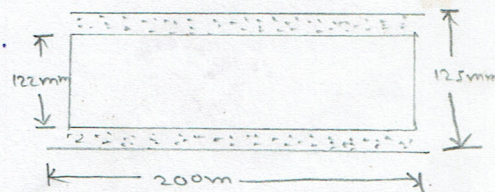
thickness (t) = $\frac{125 - 122}{2} = 1.5\text{mm}$.

$S = 0.9$.

$S = 0.9 \times 1000 = 900\text{kg/m}^3$.

$\mu = \nu S = 0.027\text{Pa.s}$.

Now; $\tau = \mu \cdot \frac{du}{dy} = 0.027 \times \frac{1}{1.5 \times 10^{-3}} = 18\text{Pa}$.



311Bha

$$\text{Area (A)} = 2\pi r l = 2 \times \pi \times 61 \text{ mm} \times 200 \text{ mm} = 0.0767 \text{ m}^2$$

$$\text{Force (F)} = T \times A$$

$$= 1.8 \times 0.0767$$

$$= 1.38 \text{ N}$$

$\therefore$ The required force is 1.38 N

27. The viscosity of a fluid is to be measured using a viscometer constructed of two 750 mm long concentric cylinder. The outer diameter of the inner cylinder is 150 mm and the gap between the two cylinders is 1.2 mm. The inner cylinder is rotated at 2007 rpm and the torque is measured to be 10 Nm. Derive a general expression for the viscosity of a fluid using the type of viscometer, and determine the viscosity of the fluid for the experiment above.

Solution;

Given;

$$\text{length (l)} = 750 \text{ mm} = 750 \times 10^{-3} \text{ m}$$

$$\text{thickness (t)} = 1.2 \text{ mm} = 1.2 \times 10^{-3} \text{ m}$$

$$\text{frequency (f)} = 2007 \text{ rpm} = \frac{2007}{60} \text{ rps}$$

$$\therefore \omega = 2\pi f = \frac{4007\pi}{60} = \frac{207\pi}{3} \text{ rad/s}$$

$$\text{Torque (T)} = 10 \text{ Nm}$$

we have,

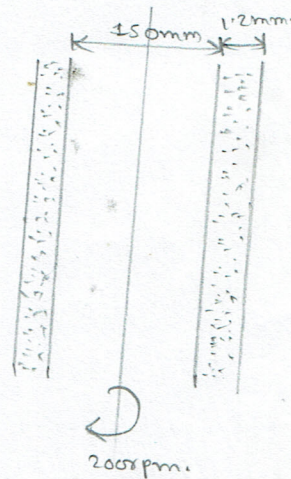
$$T = \mu \cdot \frac{dv}{dt}$$

$$\mu \cdot \frac{dv}{dt} = \frac{T}{A}$$

$$\mu \cdot \frac{F}{A} = \frac{T \cdot t}{A r^2 \omega} = \frac{T \cdot t}{2\pi r^3 l \omega}$$

$$= \frac{10 \times 1.2 \times 10^{-3}}{2\pi (75 \times 10^{-3})^3 \times 750 \times 10^{-3} \times 20.944}$$

$$\therefore \mu = 0.288 \text{ Pa}\cdot\text{s}$$



311bha

28. Two large plane surfaces are 2.4 cm apart. The space between the surface is filled with glycerine having viscosity $8.1 \times 10^{-1} \text{ N s/m}^2$. What force is required to drag a thin plate of surface area 0.5 m^2 between that large plane surface at speed of 0.6 m/sec , if the thin plate is a distance of 0.8 from one of the plane surfaces.

Solution;

Given; Viscosity of glycerine (μ) = $8.1 \times 10^{-1} \text{ N s/m}^2$.

$$\text{Area (A)} = 0.5 \text{ m}^2.$$

$$t_1 = 0.8 \times 10^{-2} \text{ m}.$$

$$t_2 = 1.6 \times 10^{-2} \text{ m}.$$

Now;

$$\tau = \mu \left[\frac{u_1}{t_1} + \frac{u_2}{t_2} \right].$$

$$= 8.1 \times 10^{-1} \left[\frac{0.6}{0.8 \times 10^{-2}} + \frac{0.6}{1.6 \times 10^{-2}} \right].$$

$$= 91.125 \text{ Pa}.$$

$$\therefore \text{Force (F)} = \tau \times A$$

$$= 91.125 \times 0.5 = 45.56 \text{ N}.$$

$\therefore$ The required force is 45.56 N .

29. A sliding fit cylinder body of diameter 14.9 cm and length 15 cm of 1 kg mass drops vertically down at a constant velocity of 5 cm/s inside a big cylinder of diameter 15 cm . They are separated by the oil. Estimate the viscosity of the oil.

Solution;

Given; velocity (u) = $5 \text{ cm/s} = 0.05 \text{ m/s}$

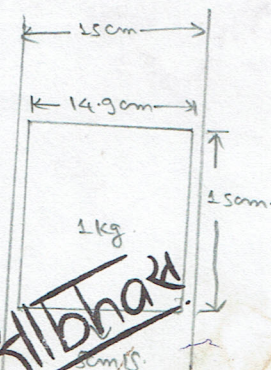
$$\text{Shear force (F)} = 1 \text{ kg} \times 9.8 \text{ m/s}^2 = 9.8 \text{ N}.$$

$$\text{Now; Area (A)} = 2\pi r l = 2\pi \left(\frac{14.9}{2} \right) \times 10^{-2} \times 15 \times 10^{-2} \text{ m}^2.$$

$$= 0.070 \text{ m}^2.$$

$$\text{Shear stress (}\tau\text{)} = \frac{F}{A} = \frac{9.8}{0.070} = 139.71 \text{ Pa}.$$

$$\text{Thickness (t)} = \frac{(15 - 14.9)}{2} \text{ cm} = 0.05 \text{ cm} = 5 \times 10^{-4} \text{ m}.$$



$$\tau = \mu \frac{du}{dy}$$

$$\text{or, } \tau = \mu \frac{u}{t}$$

$$\text{or, } 139.71 = \frac{\mu \times 0.5}{5 \times 10^{-4}}$$

$$\therefore \mu = 1.397 \text{ Pa}\cdot\text{s}$$

$\therefore$ The viscosity of the oil is $1.397 \text{ Pa}\cdot\text{s}$.

30. The dynamic viscosity of oil used for lubrication between a shaft and sleeve is $0.6 \text{ Pa}\cdot\text{s}$. The shaft is of 0.4 m diameter and rotates at 190 rpm . If the thickness of oil film is 1.5 mm and sleeve length of 90 mm , calculate the torque necessary to rotate.

Solution;

Given;

Dynamic viscosity of oil (μ) = $0.6 \text{ Pa}\cdot\text{s}$

Frequency (N) = 190 rpm .

$$\text{Now, } \omega = 2\pi \times \frac{190}{60} = 19.897 \text{ s}^{-1}$$

$$\text{Velocity } (V) = r\omega = 0.2 \times 19.897 = 3.979 \text{ m/s}$$

$$\text{Given, thickness } (t) = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$\text{Now, Shear stress } (\tau) = \mu \frac{du}{dy}$$

$$= \mu \frac{V}{t} = \frac{0.6 \times 3.979}{1.5 \times 10^{-3}} = 1591.6 \text{ Pa}$$

$$\text{Area, } A = 2\pi rl = 2\pi \times 0.2 \times 90 \times 10^{-3} = 0.113 \text{ m}^2$$

$$\text{Force} = \tau \times A$$

$$= 1591.6 \times 0.113$$

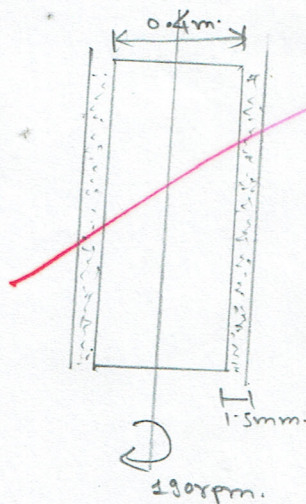
$$= 179.85 \text{ N}$$

$$\text{Torque} = F \times r = 179.85 \times 0.2$$

$$= 35.97 \text{ Nm}$$

$$= 3.571 \text{ kg Fm}$$

$\therefore$ The required torque is 3.571 kg Fm .



311bhar

21. A cylinder of 12cm radius rotates concentrically inside of a fixed cylinder of 13cm radius. Both the cylinder are 0.3m long. Determine the viscosity of the liquid, which fills the space between the cylinders. If a torque of 0.88Nm is required to maintain an angular velocity of 22 rad/s.

Solution;

$$\text{Torque (T)} = 0.88 \text{ Nm}$$

$$\text{Angular velocity } (\omega) = 22 \text{ s}^{-1}$$

$$\text{radius (R)} = 12 \text{ cm} = 12 \times 10^{-2} \text{ m}$$

Now,

$$\text{Shear force} = \frac{T}{r} = \frac{0.88}{12 \times 10^{-2}} = 7.33 \text{ N}$$

$$\text{Area (A)} = 2\pi r l = 2\pi \times 12 \times 10^{-2} \times 0.3 = 0.226 \text{ m}^2$$

$$\text{Shear stress (T)} = \frac{\text{shear force}}{\text{Area}} = 32.43 \text{ Pa}$$

$$\text{Velocity (v)} = r\omega = 12 \times 10^{-2} \times 22 = 0.754 \text{ m/s}$$

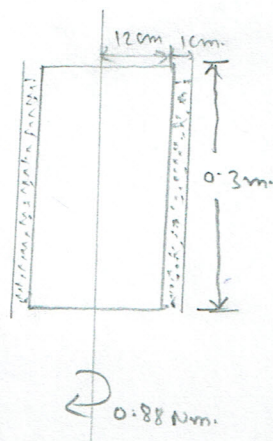
$$\text{Given, thickness (t)} = 1 \text{ cm} = 0.01 \text{ m}$$

$$T = \mu \cdot \frac{dv}{dy} = \mu \cdot \frac{v}{t}$$

$$\text{or, } 32.43 = \mu \times \frac{0.754}{0.01}$$

$$\therefore \mu = 0.43 \text{ Pa.s}$$

$\therefore$ The viscosity of the liquid is 0.43 Pa.s.



311bhan

32.

A uniform film of oil 0.10 mm thick separates two discs of 400 mm in diameter, maintained co-axially. Ignoring the edge effects, calculate the torque required to rotate one disc relative to other at the speed of 10 rev per sec. If the oil has viscosity of 0.13 Pa.s.

Solution;

$$\text{thickness } (t) = 0.1 \text{ mm.}$$

$$\text{Angular velocity } (\omega) = 10 \times 2\pi \text{ s}^{-1} \\ = 62.83 \text{ s}^{-1}.$$

$$\text{viscosity } (\mu) = 0.13 \text{ Pa.s.}$$

$$\text{Velocity } (v) = r\omega = 62.83 r.$$

$$\text{we have; } T = \mu \cdot \frac{dv}{dy},$$

$$\text{or, } T = \mu \cdot \frac{v}{t}.$$

$$\text{or, } \frac{dF}{dA} = \frac{\mu \omega r}{t}.$$

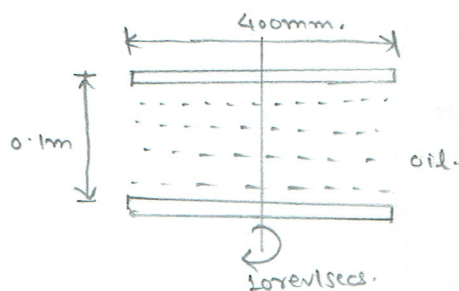
$$\text{or, } dF = \frac{\mu \omega}{t} \cdot 2\pi r^2 \cdot dr.$$

$$\text{or, } dT = \frac{2\pi \mu \omega}{t} \cdot r^3 \cdot dr.$$

$$\text{or, } T = \frac{2\pi \mu \omega}{t} \int_0^{0.2} r^3 \cdot dr.$$

$$\therefore T = 205.28 \text{ KNm.}$$

$\therefore$ The required total is 205.28 KNm.



311bhan

33. Calculate the power absorbed by fluid friction in a thrust bearing consisting of a flat disc 10cm in diameter placed at the lower end of a vertical shaft. The oil film is 0.25mm thick and the viscosity of the oil is 1.3 Poise. The shaft rotates at 2000rpm.

Given;

Solution;

$$\text{Viscosity } (\mu) = 1.3 \text{ Poise} = 0.13 \text{ Pa}\cdot\text{s}$$

$$\text{Angular velocity } (\omega) = \frac{2\pi \cdot 2000}{60} = 209.44 \text{ rad/sec.}$$

$$\text{Diameter } (d) = 0.10 \text{ m.}$$

$$\text{Thickness } (t) = 0.25 \text{ mm.}$$

Now,

$$T = \mu \frac{du}{dy}$$

$$\tau \frac{dF}{dA} = \frac{\mu \cdot \omega \cdot r}{t}$$

$$\tau \cdot dF = \frac{\mu \cdot \omega}{t} \cdot 2\pi r^2 \cdot dr$$

$$\tau \cdot dt = \frac{\mu \cdot \omega}{t} \cdot 2\pi r^3 \cdot dr$$

$$\tau \int_A dt = \frac{2\pi \mu \omega}{t} \int_0^{0.05} r^3 \cdot dr \quad [r = 0.05 \text{ m}]$$

$$\therefore T = 1.069 \text{ Nm.}$$

$$\text{Power} = T\omega = 1.069 \times 209.44 = 223.93 \text{ watt.}$$

$\therefore$ The Power absorbed by fluid friction is 223.93 watt.

Bibha

34. A conical thrust bearing idealized as cone of semi-angle 30° , maximum cone diameter 20cm rests and revolves over a uniform fluid layer of thickness 0.1cm at 600rpm. If the fluid has a viscosity of 1 poise. Calculate the heat dissipated by the bearing.

Solution;

$$\text{Viscosity } (\mu) = 1 \text{ poise} = 0.1 \text{ Pa}\cdot\text{s}$$

$$\text{thickness } (t) = 0.1 \text{ cm} = 1 \times 10^{-3} \text{ m.}$$

$$\text{Angular velocity } (\omega) = \frac{2\pi \times 600}{60} = 20\pi \text{ rad/s.}$$

Now;

$$dA = 2\pi r \cdot ds$$

$$= \frac{2\pi r}{\sin \theta} \cdot dr$$

We have;

$$T = \mu \cdot \frac{du}{dy}$$

$$\text{or, } \frac{dF}{dr} = \frac{\mu \cdot r \cdot \omega}{t} \cdot \frac{2\pi r}{\sin \theta} \cdot dr$$

$$\text{or, } dF = \frac{2\pi \mu \cdot r^2 \omega}{t \sin \theta} \cdot dr$$

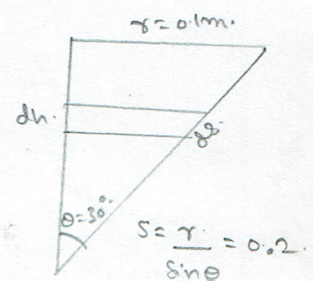
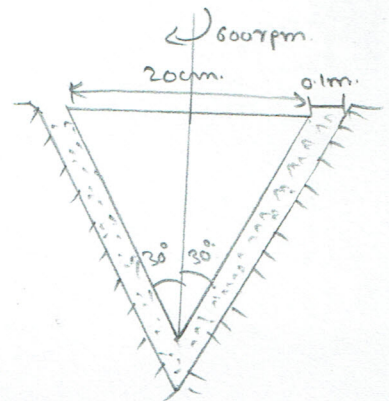
$$\text{or, } dT = \frac{2\pi \mu \cdot \omega r^3 \cdot dr}{t \sin \theta}$$

$$\text{or, } \int_A dT = \frac{2\pi \mu \omega}{t \sin \theta} \int_0^{0.1} r^3 dr$$

$$\text{or, } T = 20\pi^2 \times 0.01 = 0.02\pi^2$$

$$\therefore \text{Heat dissipated} = T\omega = 0.02\pi^2 \times 20\pi = 4\pi^3 \omega$$

$\therefore$ The heat dissipated by the bearing is $4\pi^3 \omega$.



311Bhanu

35. A conical thrust bearing idealized as cone of semi-angle 45° , with inclined length 8cm rests and revolves over a uniform layer oil thickness 0.2mm at 2000rpm. If the oil has a viscosity of 0.1 N s/m^2 , calculate the torque needed to rotate the cone.

Solution;

$$\text{viscosity } (\mu) = 0.1 \text{ Pa.s.}$$

$$\text{thickness } (t) = 0.2 \text{ mm} = 2 \times 10^{-4} \text{ m.}$$

$$\text{length } (L) = 8 \text{ cm} = 0.08 \text{ m.}$$

Now;

$$dA = 2\pi r \cdot ds = \frac{2\pi r}{\sin \theta} \cdot dr.$$

$$\omega = \frac{2\pi \times 2000}{60} = 209.44 \text{ rad/s.}$$

Now;

$$T = \int r \cdot \frac{dF}{dy} \cdot dy$$

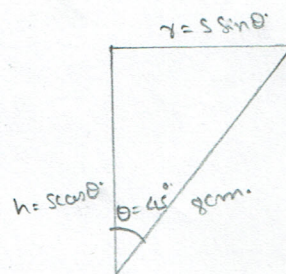
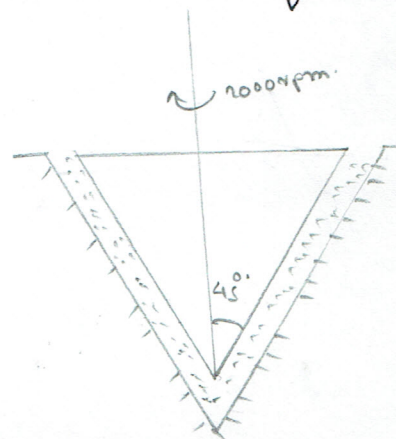
$$\text{or } \frac{dF}{dA} = \frac{\mu \cdot r \omega}{t}$$

$$\text{or } dF = \frac{\mu \omega}{t} \cdot \frac{2\pi r^2 dr}{\sin \theta}$$

$$\text{or } dT = \frac{2\pi \mu \omega}{t \sin \theta} \cdot r^3 \cdot dr.$$

$$\text{or } T = \int_A dT = \frac{2\pi \mu \omega}{t \sin \theta} \int_0^{0.08 \sin 45^\circ} r^3 \cdot dr.$$

$$\therefore T = 2.38 \text{ Nm}$$



ADY9112

311bhar

Tutorial 2

Pressure and Head

by
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- 1) What depth is necessary in a liquid to produce a pressure of 250 kPa, if the specific gravity is a) 1.0 b) 0.8 c) 13.6 d) 1.59 and e) 0.68?

Solution;

Given, Pressure (P) = 250 kPa = 250×10^3 Pa.

For specific gravity = 1.0.

$$P = \gamma h.$$

$$\text{or, } 250 \times 10^3 = 1 \times 9810 \times h.$$

$$\therefore h = 25.5 \text{ m.}$$

For specific gravity = 0.8.

$$P = \gamma h.$$

$$\text{or, } 250 \times 10^3 = 0.8 \times 9810 \times h.$$

$$\therefore h = 31.90 \text{ m.}$$

For specific gravity = 13.6.

$$P = \gamma h.$$

$$\text{or, } 250 \times 10^3 = 13.6 \times 9810 \times h.$$

$$\therefore h = 1.874 \text{ m.}$$

For specific gravity 1.59.

$$P = \gamma h.$$

$$\text{or, } 250 \times 10^3 = 1.59 \times 9810 \times h.$$

$$\therefore h = 16 \text{ m.}$$

For specific gravity 0.68.

$$P = \gamma h.$$

$$\text{or, } 250 \times 10^3 = 0.68 \times 9810 \times h.$$

$$\therefore h = 37.50 \text{ m.}$$

2. Assuming the density of air to be constant at 1.23 kg/m^3 , calculate the pressure change from the top of a mountain to its base if the elevation change is 3000 m.

Solution;

Given; Density (ρ) = 1.23 kg/m^3 .

height (h) = 3000 m.

Now,

$$P = \rho gh.$$

$$\text{or, } P = 1.23 \times 9.81 \times 3000$$

$$\text{or, } P = 36198 \text{ Pa.}$$

$$\therefore P = 36.19 \text{ kPa.}$$

$$\therefore \text{The change is } 36.19 \text{ kPa.}$$

311bha

3.

What is the pressure in KN/m^2 absolute and gauge at a point 3m below the free surface of a liquid having density $1.53 \times 10^3 \text{ kg/m}^3$, if the atmospheric pressure is equivalent to 750mm of Hg?

Solution;

Given;

$$\text{Height } (h) = 3\text{m.}$$

$$\text{Density } (\rho) = 1.53 \times 10^3 \text{ kg/m}^3$$

Now;

$$\begin{aligned} \text{Gauge Pressure} &= \rho \times g \times h \\ &= 1.53 \times 10^3 \times 9.81 \times 3 \\ &= 45.02 \text{ KN/m}^2. \end{aligned}$$

$$\text{Absolute Pressure} = P_{\text{atm}} + P_{\text{gauge}}$$

$$= 13600 \times 9.81 \times 750 \times 10^{-3} + 45.02 \times 10^3$$

$$= 100062 + 45020$$

$$= 145082 \text{ N/m}^2$$

$$= 145.08 \text{ KN/m}^2$$

$\therefore$ The absolute pressure is 145.08 KN/m^2 and gauge pressure is 45.02 KN/m^2

4.

A closed tank contains 0.5m of mercury; 2m of water 3m oil of density 600 kg/m^3 , and there is an air space above the oil. If the gauge pressure at the bottom of the tank is 200 KN/m^2 , find the pressure of air at the top of the tank.

Solution;

Given;

$$P_{\text{gauge}} = 200 \times 10^3 \text{ N/m}^2 = 2 \times 10^5 \text{ N/m}^2$$

Now;

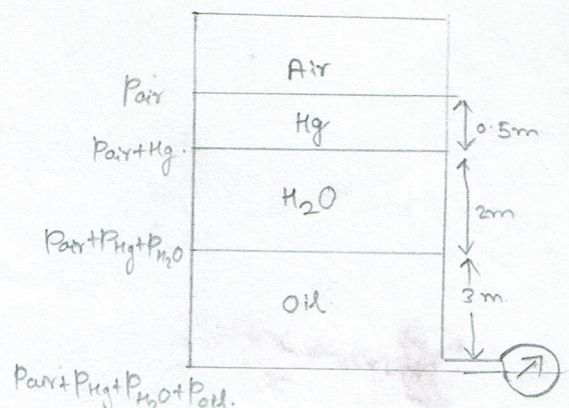
$$P_{\text{air}} + P_{\text{Hg}} + P_{\text{H}_2\text{O}} + P_{\text{oil}} = P_{\text{gauge}}$$

$$\begin{aligned} \text{or, } P_{\text{air}} &= 2 \times 10^5 - 13600 \times 9.81 \times 0.5 - 1000 \times 9.81 \times 2 \\ &\quad - 600 \times 9.81 \times 3 \end{aligned}$$

$$\text{or, } P_{\text{air}} = 96014 \text{ Pa.}$$

$$= 96.014 \text{ kPa.}$$

$\therefore$ The pressure of the air is 96.014 kPa.



311bhard

5. The hydraulic press has a diameter ratio between the two pistons of 8 to 1. The diameter of the larger piston is 600mm and it is required to support a mass of 3500 kg. The press is filled with a liquid of $S=0.8$. Calculate the force on the smaller piston.

- when the two pistons are in same level.
- when the smaller piston is 2.6m below the larger piston.

Solution;

Given;

Diameter ratio of larger piston to smaller piston = 8:1.

Diameter (d) = 600mm.

Force on larger piston = $3500 \times 9.81 = 34335 \text{ N}$.

Now, when the pistons are in same level.

$$P_{\text{large}} = P_{\text{small}}$$

$$\text{or, } \frac{F_{\text{large}}}{A_{\text{large}}} = \frac{F_{\text{small}}}{A_{\text{small}}}$$

$$\text{or, } \frac{34335}{\pi \times \left(\frac{8d}{4}\right)^2} = \frac{F_{\text{small}}}{\pi \times \frac{d^2}{4}}$$

$$\therefore F_{\text{small}} = 536.48 \text{ N}$$

When the smaller piston is 2.6m below the large piston.

$$P_{\text{small}} = P_{\text{large}} + \gamma \times 2.6$$

$$= \frac{F_{\text{large}}}{A_{\text{large}}} + 1000 \times 9.81 \times 2.6 \times 0.8$$

$$= \frac{3500 \times 9.81}{\pi \times \left(\frac{600}{4}\right)^2} + 20404$$

$$F_{\text{small}} = P_{\text{small}} \times A_{\text{small}}$$

$$= \frac{3500 \times 9.81}{\pi \times \left(\frac{600}{4}\right)^2} \times A_{\text{small}} + A_{\text{small}} \times 20404$$

$$= 3500 \times 9.81 \times \left(\frac{1}{8}\right)^2 + \pi \times \left(\frac{1}{8} \times 0.3\right)^2 \times 20404$$

$$\therefore F_{\text{small}} = 626.66 \text{ N}$$

311bhan

7. Calculate the gauge reading as shown in the figure.

Solution;

$$P_A = 0$$

Let the pressure to be determined to be P .

Then;

$$P = P_{atm} + 9810 \times 3$$

Since height are equal we have;

$$P_A = P_{A'}$$

$$\therefore P_{A'} = 0$$

$$P_{A'} = P_{atm} + P_{A'B}$$

$$\text{or, } 0 = P_{atm} + 13600 \times 0.2 \times 9.81$$

$$\therefore P_{atm} = -26683.2 \text{ Pa}$$

$$\begin{aligned} \therefore P &= -26683.2 + 3 \times 9810 \\ &= -26683.2 + 29430 \\ &= 2746.8 \text{ Pa} \end{aligned}$$

$\therefore$ The required pressure is 2746.8 Pa

- 8) Find the pressure between the two end points of the manometer arrangement as shown in the figure.

Solution;

we have;

$$P_S = (0.5 + 0.3) - 0.6 = 0.2 \text{ m}$$

Since the height of liquid in both the column are equal

$$P_S = P_R$$

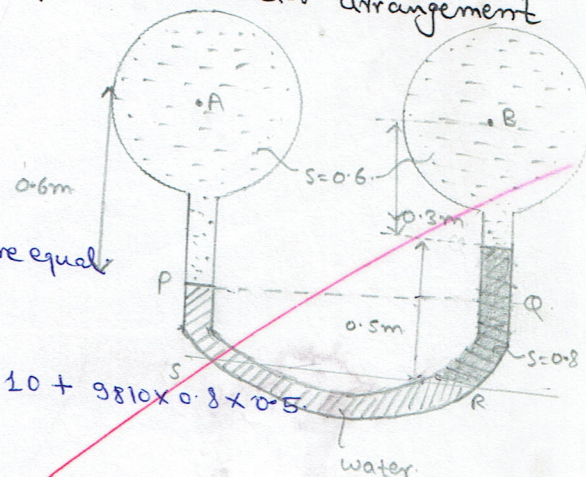
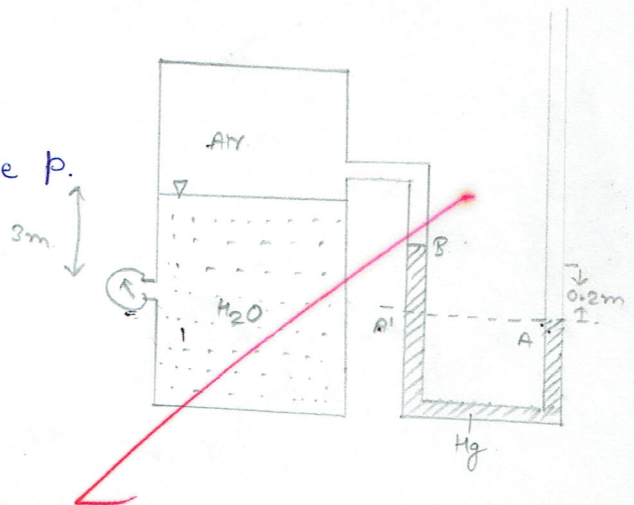
$$\text{or, } P_A + 0.6 \times 9810 \times 0.6 + 9810 \times P_S = P_B + 0.3 \times 0.6 \times 9810 + 9810 \times 0.8 \times 0.5$$

$$\text{or, } P_A + 3531.60 + 9810 \times 0.2 = P_B + 1765.80 + 3924$$

$$\text{or, } P_A - P_B = 196.20 \text{ Pa}$$

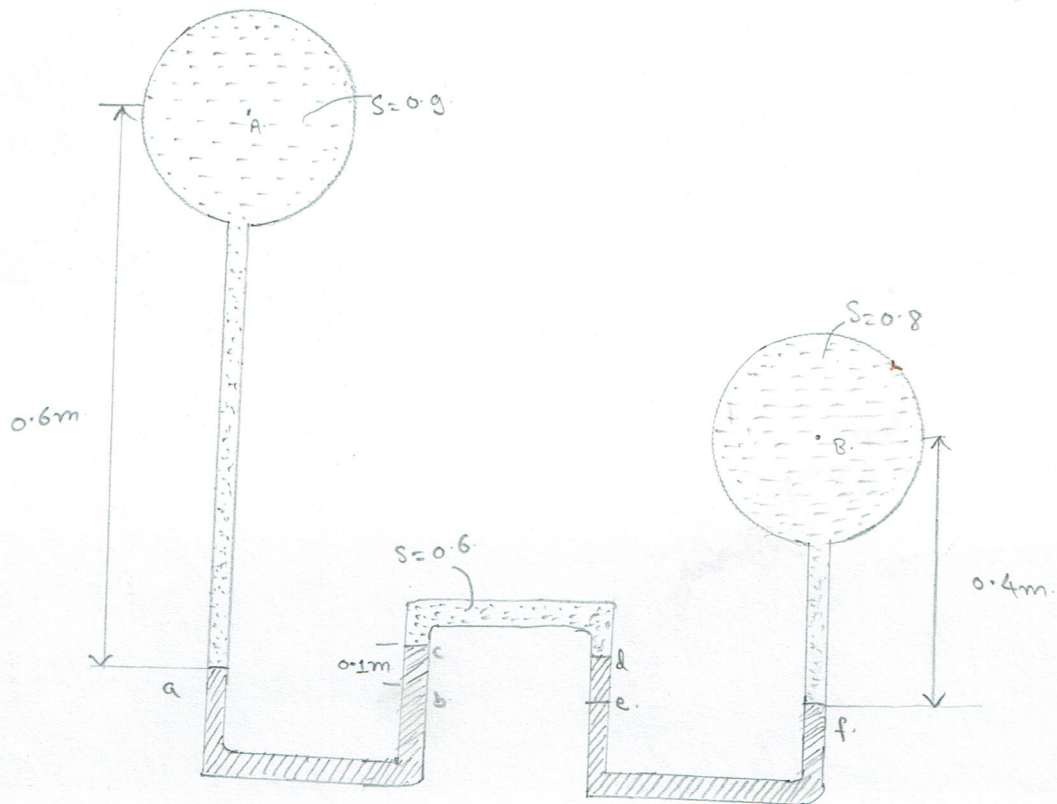
$$\therefore P_A - P_B = 196.20 \text{ Pa}$$

$\therefore$ The required pressure difference is 196.20 Pa



311Bhan

9. Find the pressure difference between the two endpoints of the manometer arrangement as shown in figure.



Solution,

$$P_c = P_d$$

and

$$P_c = P_A + 0.6 \times 0.9 \times 9810 - 0.1 \times 13.6 \times 9810$$

$$\Rightarrow P_c = P_A - 8044.2 \quad \text{--- i)}$$

And,

$$P_d = P_B + 0.4 \times 0.8 \times 9810 - 0.1 \times 13.6 \times 9810$$

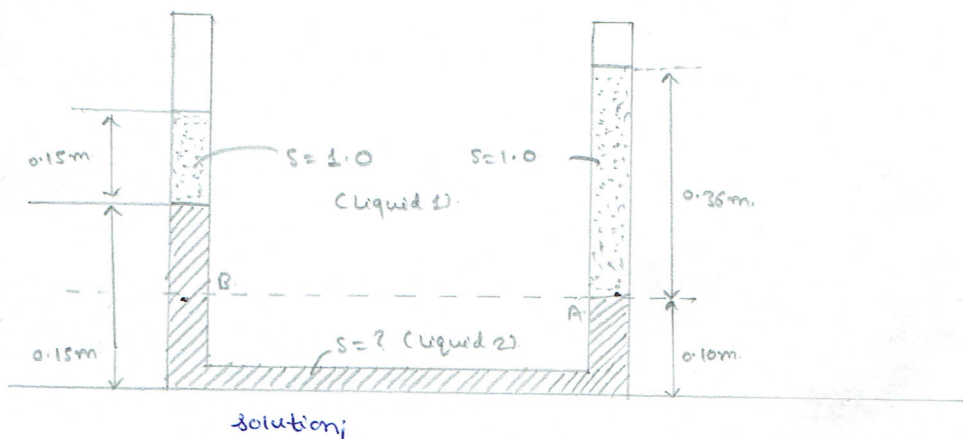
$$P_d = P_B - 10202.4 \quad \text{--- ii)}$$

From i) and ii).

$$P_A - P_B = -2158.2 \text{ Pa}$$

Alibhah

10. Find the specific gravity of the liquid at the bottom of the manometer as shown in figure.



solution;

$$P_A = P_B$$

$$\gamma_1 \cdot 0.36 \times \gamma_1 = 0.15 \times \gamma_1 + (0.15 - 0.10) \gamma_2$$

$$\gamma_1 (0.36 - 0.15) \gamma_1 = 0.05 \gamma_2$$

$$\gamma_1 \cdot 2060 \cdot 10 = 0.05 \gamma_2$$

$$\therefore \gamma_2 = 41202$$

$$\gamma_1 \cdot S \times 9810 = 41202$$

$$\therefore S = 4.2$$

$\therefore$ The required specific gravity of liquid is 4.2.

11. Find the level of the liquid in the manometer arrangement as shown in the figure.

solution; given; $P_A = 130 \times 10^3 \text{ Pa}$.

$$\text{Here; } \sin 40^\circ = \frac{h}{0.4}$$

$$\therefore h = 0.4 \sin 40^\circ$$

$$= 0.257 \text{ m}$$

we have;

$$P_b = P_a$$

$$\gamma_1 P_A + 9810 \times 0.3 = 0.9 \times 9810 \times l + (h - 0.2) \times 2 \times 9810$$

$$\gamma_1 P_A + 2943 = 0.9 \times 9810 \times l + (0.257 - 0.2) \times 2 \times 9810$$

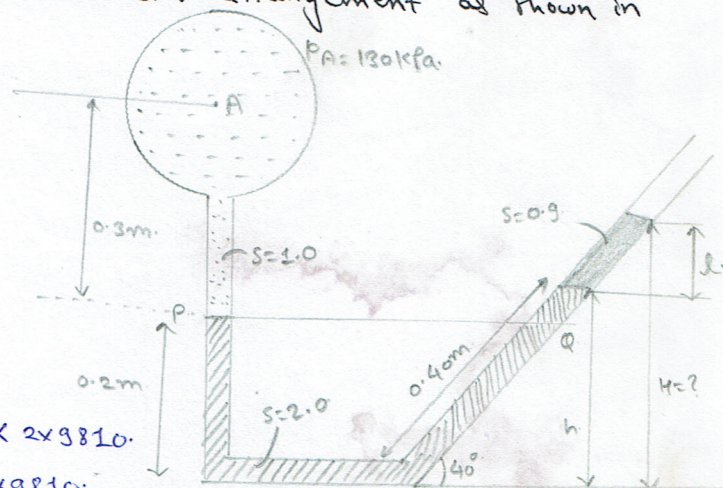
$$\gamma_1 130 \times 10^3 + 2943 = 0.9 \times 9810 \times l + 118.34$$

$$\therefore l = 14.94 \text{ m}$$

$$\therefore H = l + h$$

$$= 14.94 + 0.257$$

$$\therefore H = 15.187 \text{ m}$$



311bhan

12. Find the pressure at Point 'A' in the manometer as shown in figure below;

Solution;

we have;

$$P_p = P_q.$$

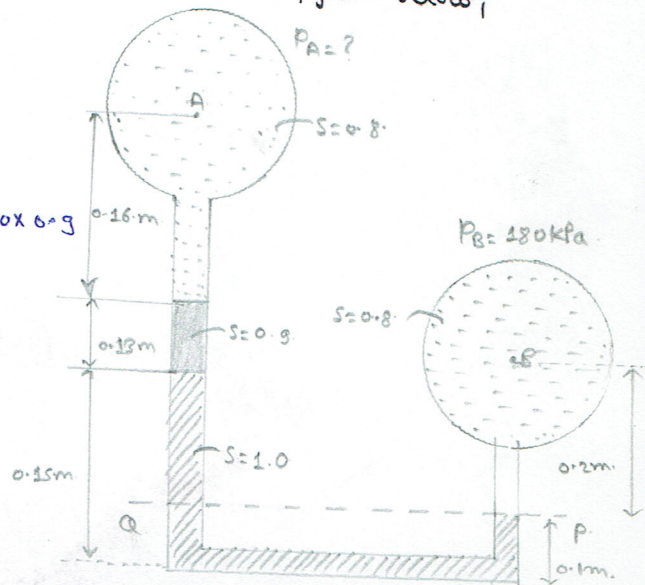
$$\therefore P_B + 0.2 \times 9810 \times 0.8 = P_A + 0.16 \times 0.8 \times 9810 + 0.13 \times 9810 \times 0.9 + (0.05) \times 9810.$$

$$\therefore P_B + 1569.60 = P_A + 2893.95$$

$$\therefore P_A = P_B - 1324.35. [\because P_B = 180 \text{ kPa}]$$

$$\therefore P_A = 178675.65 \text{ Pa.}$$

$$= 178.675 \text{ kPa.}$$



13. Calculate the force necessary to keep the level of water in the tube as shown below.

Solution;

Given, mass of piston (m) = 0.1 kg.

Radius of piston (r) = 0.1 m.

$$\text{weight (w)} = 0.1 \times 9.81 = 0.981 \text{ N.}$$

Let the additional force required be 'F'.

For equilibrium;

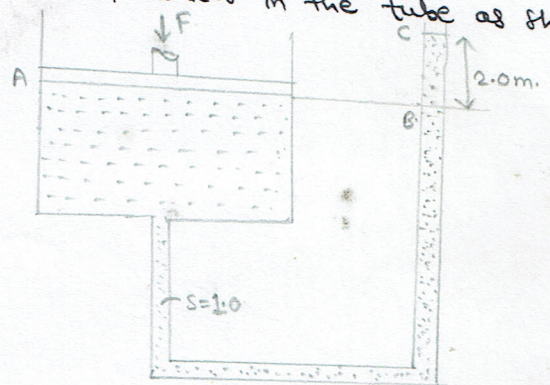
$$P_A = P_B.$$

$$\therefore \frac{F + w}{A} = 2 \times 9810$$

$$\therefore \frac{F + 0.981}{\pi \times (0.1)^2} = 19620$$

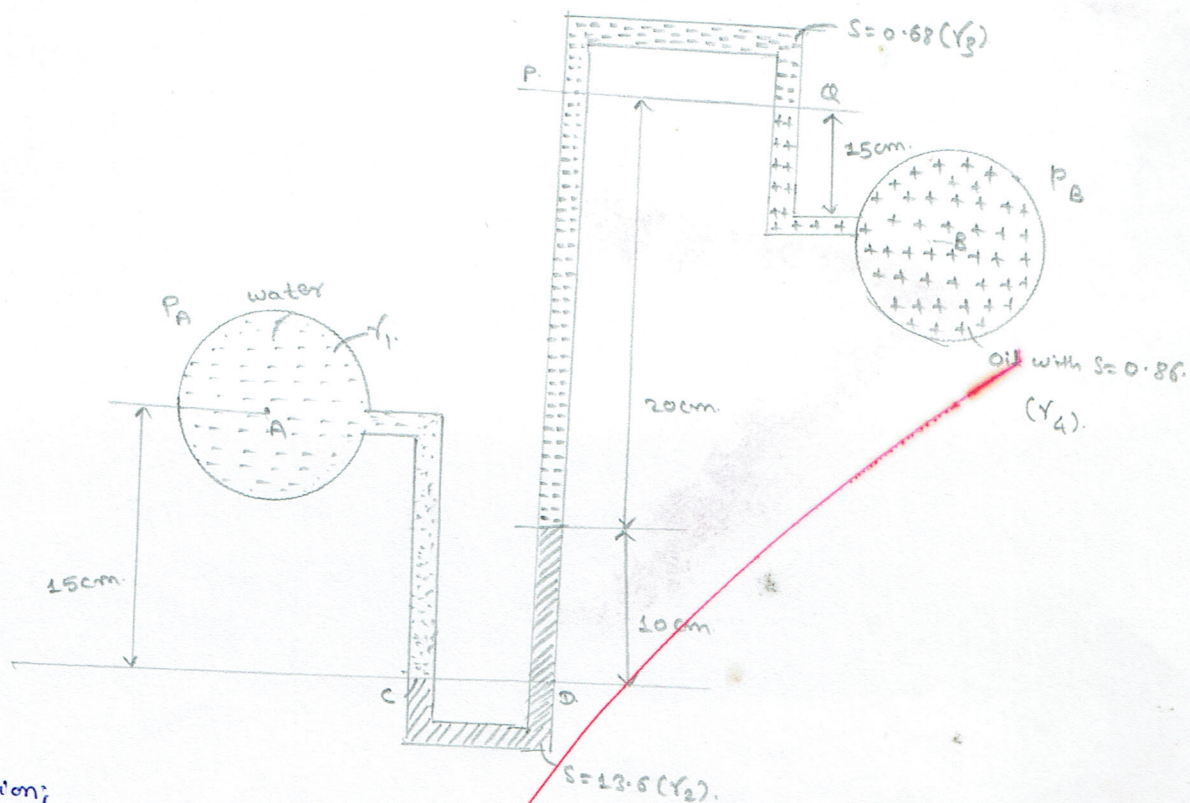
$$\therefore F + 0.981 = 616.62 \text{ N.}$$

$$\therefore F = 615.39 \text{ N.}$$



Albhan

14. Determine the pressure difference between the water pipe and the oil pipe as shown in the figure below.



Solution;

we have;

$$P_C = P_D.$$

$$0.1P_A + 0.15 \times Y_1 = Y_2 \times 0.1 + 0.2 \times Y_3 + P_p.$$

$$\sigma_1 p_A + 0.15 \times 9810 = 133416 \times 0.1 + 0.2 \times 6670.8 + p_p$$

$$\sigma_1 P_A + 1471.5 = 14675.76 + P_B - \sigma_2$$

And;

$$p_p = p_q$$

$$= P_R - 8436.6 \times 0.15,$$

$$= P_R - 1265.4 \text{ g.}$$

But $P_R = P_B$. [By Pascal's Pressure distribution law].
Then; $p = h \rho g$

Then; $p_p = p_B - 1265.49$.

From a),

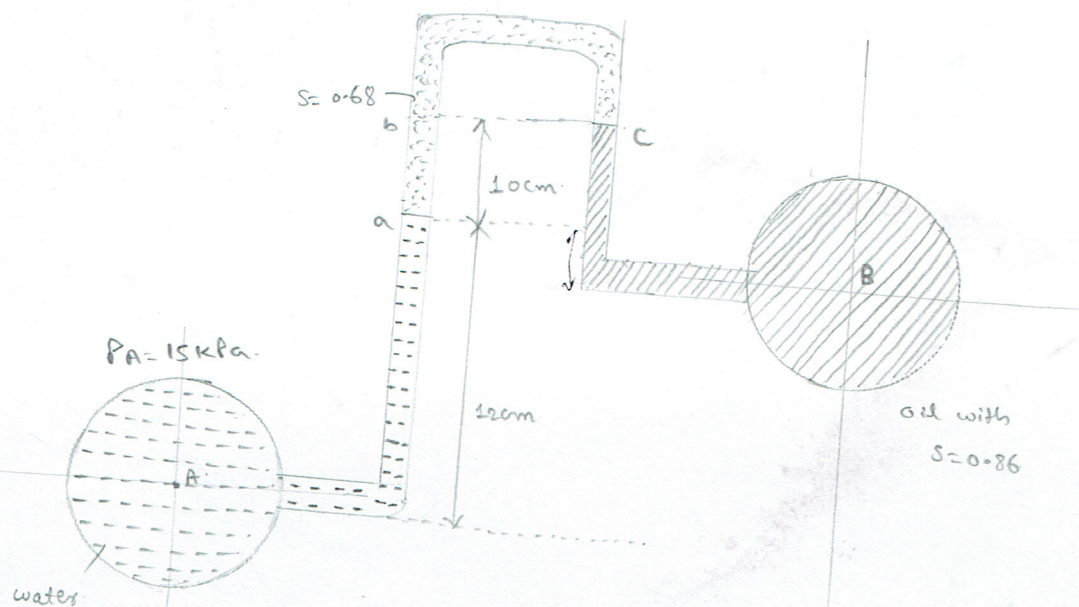
$$P_A = 13204.26 + P_B = 1265.49.$$

$$\therefore p_A - p_B = 11938.77 \text{ Pa}$$

$$= 11.938 \text{ KPa}$$

9. अल्लभार्त

15. What is the pressure in the oil pipe as shown in figure below, if the pressure in the water pipe is 15 kPa?



Solution;

$$P_b = P_A - 0.12 \times 9810 - 0.1 \times 0.68 \times 9810$$

$$\therefore P_b = P_A - 1844.28$$

And

$$P_b = P_c$$

$$P_c = P_B - 0.1 \times 0.86 \times 9810$$

$$\text{or, } P_A - 1844.28 = P_B - 843.67$$

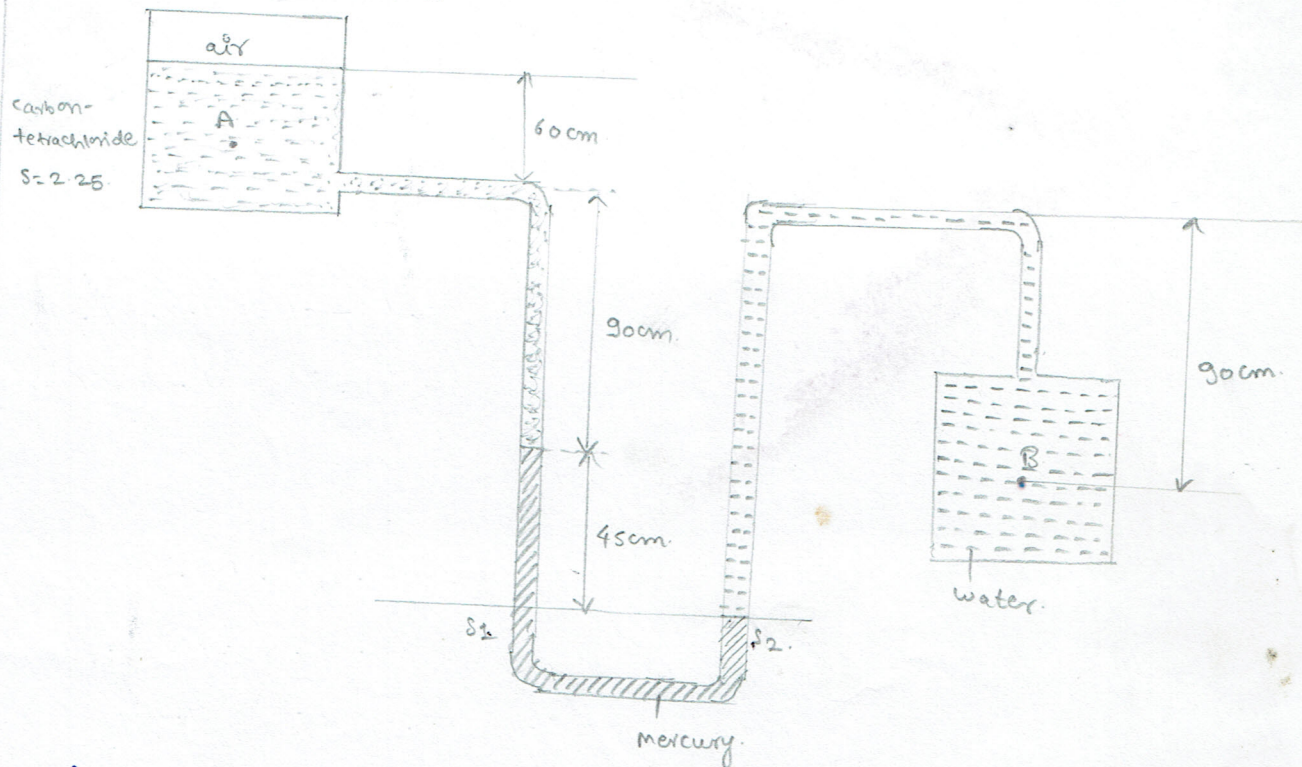
$$\text{or, } 15000 - 1844.28 + 843.67 = P_B$$

$$\therefore P_B = 13999.5 \text{ Pa}$$

Albhar

16.

A closed tank A is filled with Carbon tetrachloride with $S = 2.25$ and has a small air space at its top as shown in the figure below. A manometer is connected between the tank A and a water tank as shown. If the pressure at Point B in the water tank is 1.2 kgf/cm^2 , find the pressure in KN/m^2 in the closed air space at the top of the tank A.



Solution;

we have;

$$P_{S_1} = P_{S_2}$$

$$\text{or, } P_{S_2} = \gamma_{Hg} \times 0.45 + 2.25 \gamma_{H_2O} \times 0.9 + 2.25 \gamma_{H_2O} \times 0.6 + P_{air}$$

$$\Rightarrow P_{S_2} = 93145.95 + P_{air}$$

Also,

$$P_{B1} = P_{S_2} - \gamma_{H_2O} \times 0.45$$

$$\text{But } P_{B1} = P_B = \frac{1.2 \times 9.81}{10^{-4}} = 117720 \text{ N/m}^2$$

$$\therefore P_{S_2} = 117720 + \gamma_{H_2O} \times 0.45$$

$$= 122134.5 \text{ N/m}^2$$

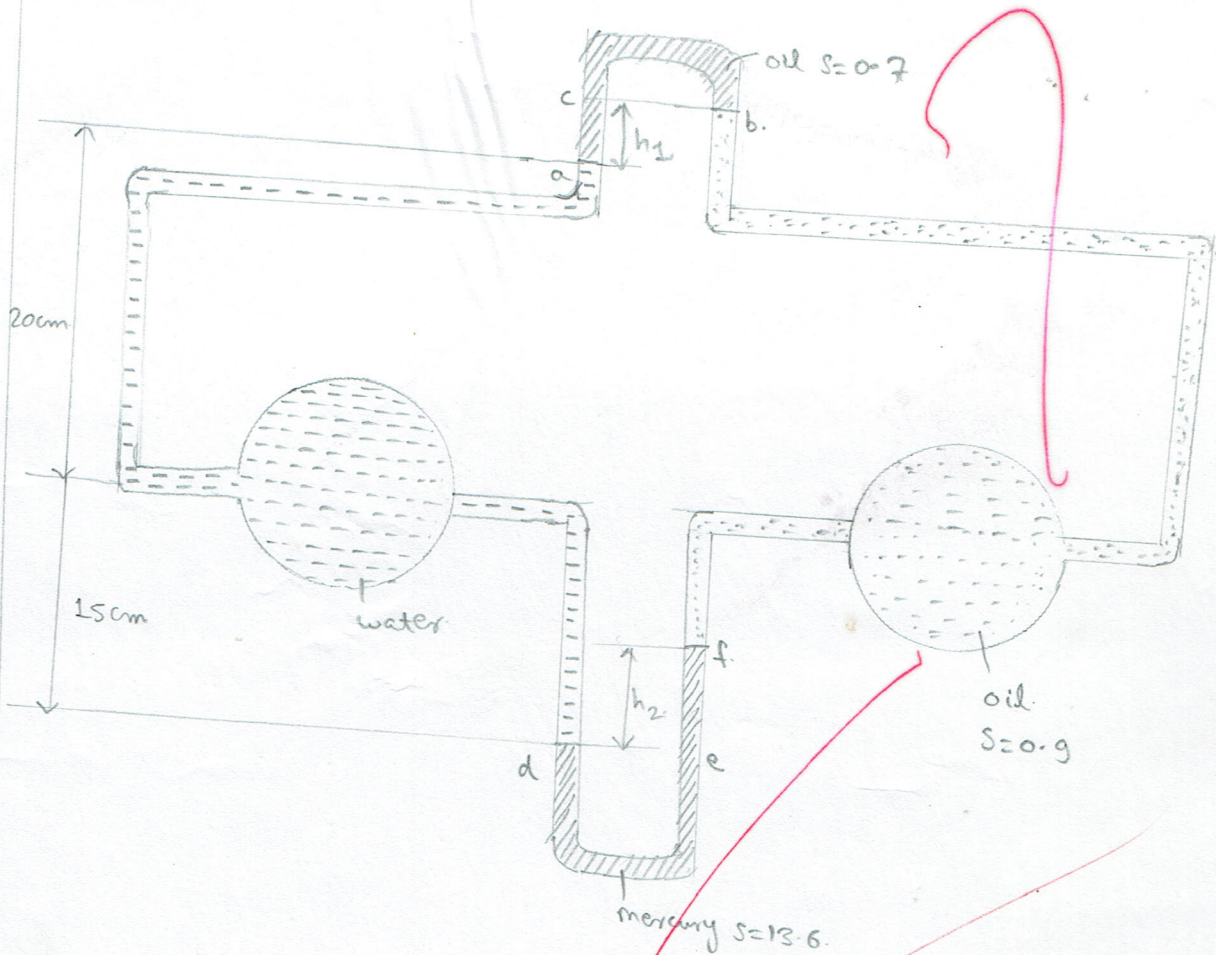
$$\text{Thus, } P_{air} = P_{S_2} - 93145.95$$

$$\text{or, } P_{air} = 28988.55 \text{ N/m}^2$$

$$\therefore P_{air} = 28.98 \text{ KN/m}^2$$

Alibhara

- 17 Two U-tube manometers are upright and the other inverted type, are connected across a water line and an oil as shown in figure. If $h_1 = 5\text{cm}$ what shall h_2 be?



Solution;

$$P_c = P_b \text{ and } P_d = P_e$$

$$P_c = P_A - 0.2 \times 9810 - 0.05 \times 0.7 \times 9810 = P_A - 2305.35 \text{ --- (i)}$$

$$P_b = P_B - 0.25 \times 0.9 \times 9810 = P_B - 2207.25 \text{ --- (ii)}$$

From (i) and (ii).

$$P_A - P_B = 98.1 \text{ --- (iii)}$$

Also;

$$P_d = P_e$$

$$\Rightarrow P_A + 0.15 \times 9810 = P_B + (0.15 - h_2) \times 0.9 \times 9810 + h_2 \times 13.6 \times 9810$$

$$\Rightarrow P_A - P_B = 1324.35 - 8829h_2 + 133416h_2 - 14715$$

$$\Rightarrow h_2 = 1.97 \times 10^{-3} \text{ m} = 0.197 \text{ cm}$$

Tutorial 3

Hydrostatics

by
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1. A culvert draws off water from the base of a reservoir. The entrance to the culvert is closed by a circular gate of 1.2m in diameter, which can be rotated about its horizontal diameter. Show that the turning moment on the gate is independent of the depth of water. If the gate is completely immersed and find the value of this moment.

Solution;

Given;

'h' is the distance from the free surface at which the pressure force 'P' acts.

Moment = $P \times (h - \bar{x})$ where $(h - \bar{x})$ is the moment arm.

Now;

$$P = \gamma A \bar{x}$$

$$h = \bar{x} + \frac{I_{CG} \sin^2 \theta}{A \bar{x}}$$

$$\text{or } h - \bar{x} = \frac{I_{CG} \times \sin^2 90^\circ}{A \bar{x}}$$

$$\therefore \text{Moment} = \frac{P \cdot I_{CG}}{A \bar{x}}$$

$$= \frac{\gamma A \bar{x} \cdot I_{CG}}{A \bar{x}}$$

$$= \gamma I_{CG}$$

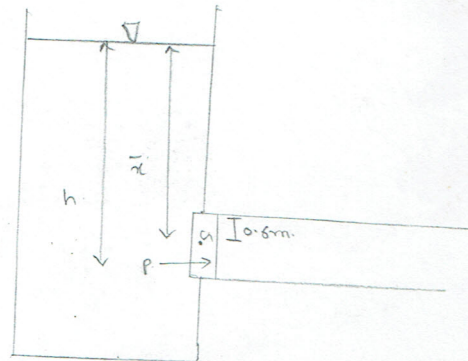
where,

I_{CG} is the moment of inertia about centroidal line parallel to the immersed plane surface and perpendicular to $\frac{h}{\sin \theta}$.

$$= \frac{784}{8} = 0.1017 \text{ m}^4$$

$$\therefore \text{Moment} = 9810 \times 0.1017$$

$$= 998.53 \text{ Nm}$$



311bhar

2. As shown in figure below a circular water gate, diameter 2m is supported by a horizontal shaft. What is the moment around the shaft to keep the water gate closed?

Solution;

Let 'h' be the height from the free surface at which the pressure force 'P' acts.

(h - $\bar{x}$) is the moment arm.

$$\therefore \text{Moment, } M = (h - \bar{x}) \times P.$$

$$h = \bar{x} + \frac{I_{CG} \sin^2 \theta \cdot \rho \cdot g \cdot A \bar{x}}{A \bar{x}}$$

$$\text{or, } h - \bar{x} = \frac{I_{CG} \sin^2 \theta \cdot \rho \cdot g}{A \bar{x}}$$

$$\therefore M = \frac{I_{CG}}{A \bar{x}} \times P.$$

$$\therefore P = \gamma A \bar{x}$$

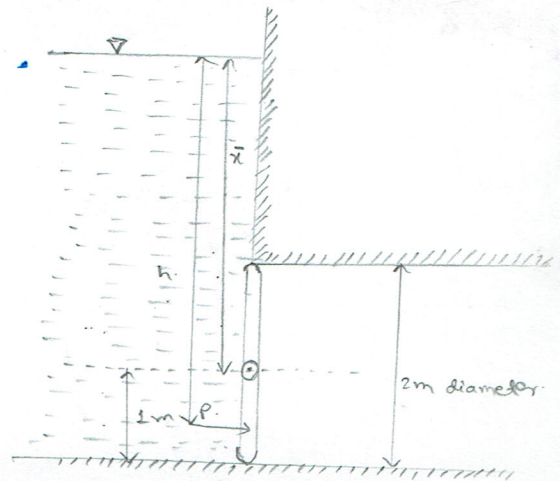
$$M = \frac{I_{CG} \times (A \times \gamma \bar{x})}{A \bar{x}}$$

$$= I_{CG} \times \gamma.$$

Here,

$$I_{CG} = \frac{\pi r^4}{4} = \frac{\pi}{4} = 0.7857 \text{ m}^4.$$

$$\therefore \text{Moment (m)} = 0.7857 \times 9810 \\ = 7707.85 \text{ Nm}.$$



3. A cubical tank has side of 1.5m. It contains water for the lower 0.6m depth. The upper remaining part is filled with oil of $S = 0.9$. Calculate for one vertical side of the tank:
- Total Pressure.
 - Position of centre of pressure. Also draw pressure diagram.

Solution;

Given;

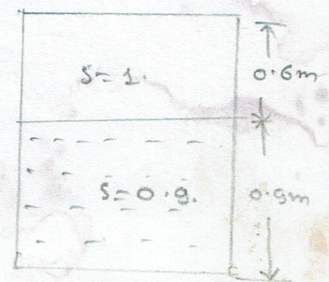
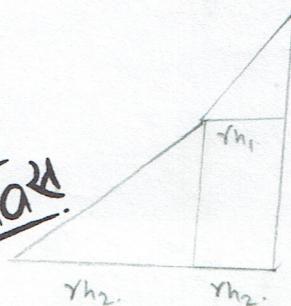
Depth of water (h_1) = 0.6m

Depth of oil (h_2) = 1.5 - 0.6 = 0.9m

$S_{oil} = 0.9$.

Now;

311Bhard



Now,

$$\begin{aligned}\text{Total pressure} &= \frac{1}{2} \times \gamma \times 0.9^2 \times 0.9 \times 1.5 + \frac{1}{2} \times \gamma \times 0.6^2 \times 0.6 \times 1.5 + \gamma \times 0.9^2 \times 0.6 \times 1.5 \\ &= 5363.6175 + 2648.70 + 7151.49 \\ &= 15163.8075 \text{ kN} \\ &= 1545.75 \text{ kgf}.\end{aligned}$$

Now, for C.G.,

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$\begin{aligned}&= \frac{\frac{1}{2} \times \gamma \times 0.9^2 \times 0.9 \times (0.6 + 0.3) + \frac{1}{2} \times \gamma \times 0.6 \times 0.6 \times (0.2) + \gamma \times 0.9^2 \times 0.6 \times 0.3}{10109.205}\end{aligned}$$

$$= \frac{5001.6285}{10109.205}$$

$$10109.205$$

$$= 0.4947 \text{ m (from bottom)}.$$

31/10/2021

4. If a triangle height 'd' and base 'b' is vertical and submerged in a liquid with its base at the liquid surface, derive an expression for the depth to its centre of pressure.

Solution;

Given,

'd' be the height of the triangle.

'b' is the base of the triangle.

Let the centroid of the triangle be at $\bar{x}$ from the free surface. Then;

$$\bar{x} = \frac{1}{3} \times DB = \frac{1}{3} \times d.$$

Moment of inertia about centroidal axis for $\triangle ADB$ is;

$$I_A = \frac{b}{2} \frac{d^3}{36}.$$

Similarly, MI about centroidal axis for $\triangle BDC$ is;

$$I_B = \frac{b}{2} \frac{d^3}{36}.$$

$$\therefore \text{Total moment of inertia (MI)} = I_A + I_B \\ = \frac{bd^3}{36}.$$

We know;

$$h = \bar{x} + \frac{I}{A\bar{x}}$$

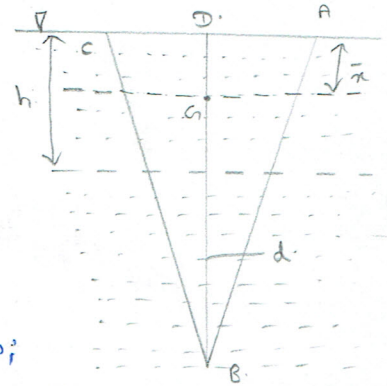
$$= \frac{d}{3} + \frac{bd^3}{36 \times \frac{1}{2} \times b \times d \times \frac{1}{3} \times d}.$$

$$= \frac{d}{3} + \frac{d}{6}.$$

$$= \frac{3d}{6}.$$

$$= \frac{d}{2}.$$

$$= 0.5d.$$



311bhan

5. A circular area of diameter 'd' is vertical and submerged in a liquid. Its upper edge is coincident with the liquid surface. Derive an expression for the depth to its centre of pressure.

Solution;

Let 'C' be the centre of a circle.

Given, diameter = d.

MI about centroidal axes;

$$I_{CG} = \frac{\pi r^4}{4} = \frac{\pi \times d^4}{16 \times 4} = \frac{\pi d^4}{64}$$

Let 'h' be the depth of pressure force from the free surface.

Then;

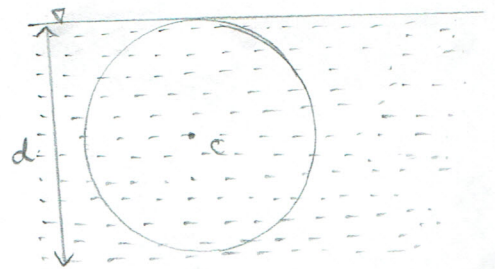
$$h = \bar{x} + \frac{I_{CG}}{A\bar{x}}$$

$$= \frac{d}{2} + \frac{\pi d^4}{64 \times \pi \times \frac{d^2}{4} \times \frac{d}{2}}$$

$$= \frac{d}{2} + \frac{d}{8}$$

$$= \frac{4d + d}{8}$$

$$\therefore h = \frac{5d}{8}$$



6. Find the magnitude and point of application of the force on the circular gate shown in figure.

Solution;

$$\bar{x} = 2 + r \sin 60^\circ$$

$$= 2 + 0.8 \sin 60^\circ$$

$$= 2.69 \text{ m}$$

$$\text{Area} = \pi r^2$$

$$= \frac{\pi}{4} \times \left(\frac{1.6}{2}\right)^2$$

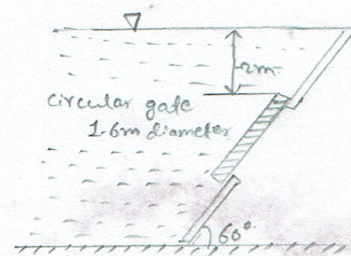
$$= 2.0114$$

$$I_{CG} = \frac{\pi r^4}{4} = \frac{\pi \times (0.8)^4}{28} = 0.3218 \text{ m}^4$$

Then;

$$h = \bar{x} + \frac{I_{CG} \sin^2 60^\circ}{A\bar{x}}$$

$$= 2.69 + \frac{0.3218 \times 3}{2.0114 \times 2.69} = 2.73 \text{ m}$$



311bha
F. bhash
F. charya

$$\begin{aligned}\text{Total Pressure (P)} &= \gamma A \bar{x} \\ &= 9810 \times 2.69 \times 2.0114 \\ &= 53078.63 \text{ N} \\ &= 53.07 \text{ kN}\end{aligned}$$

7. The top of the each gate as shown in the figure below lies 4m below the water surface. Find the location and magnitude of the force acting on one side assuming vertical orientation.

a. Solution;
Area of a surface = πr^2 .

$$\begin{aligned}&= \frac{22}{7} \times 4 \\ &= 12.57 \text{ m}^2.\end{aligned}$$

$$\bar{x} = 4 + 2 = 6 \text{ m}$$

$$\begin{aligned}P &= \gamma A \bar{x} \\ &= 9810 \times 12.57 \times 6 \\ &= 739954.28 \text{ N} \\ &= 739.54 \text{ kN}\end{aligned}$$

Let 'h' be the location of Pressure force then;

$$\begin{aligned}h &= \bar{x} + \frac{I_{\text{CG}}}{A \bar{x}} \\ &= 6 + \frac{7 \times 2^4}{4 \times 7 \times 2^2 \times 6}\end{aligned}$$

$$= 6.167 \text{ m}.$$

b.

$$\begin{aligned}\text{Area of a surface} &= \frac{\pi r^2}{2} \\ &= \frac{22}{7} \times 2 \\ &= 6.2857 \text{ m}^2.\end{aligned}$$

$$\bar{x} = 6 \text{ m}.$$

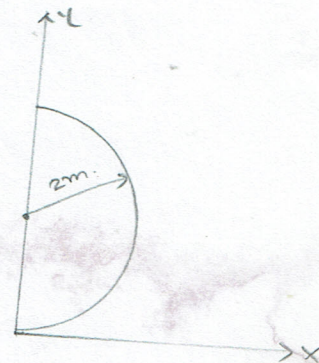
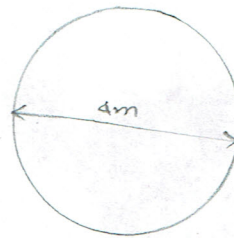
$$\begin{aligned}P &= \gamma A \bar{x} \\ &= 9810 \times 6.2857 \times 6 \\ &= 369977.14 \text{ N} \\ &= 369.977 \text{ kN}\end{aligned}$$

Let 'h' be the location of Pressure force then;

$$h = \bar{x} + \frac{I_{\text{CG}}}{A \bar{x}} = 6 + \frac{7 \times 2^4}{8 \times 7 \times 2^2 \times 6} = 6.167 \text{ m}.$$

$$x_p = \frac{4r}{3\pi} = \frac{4 \times 2 \times 7}{3 \times 22} = 0.8488.$$

$$\therefore (x_p, h) = (0.8488, 6.167).$$



Altohar

c).

$$\text{Area of the gate (A)} = \frac{1}{2} \times 3 \times 4 = 6 \text{ m}^2.$$

$$\bar{x} = 4 + \frac{4}{3} = 5.33 \text{ m.}$$

$$F = \gamma A \bar{x}$$

$$= 9810 \times 6 \times 5.33$$

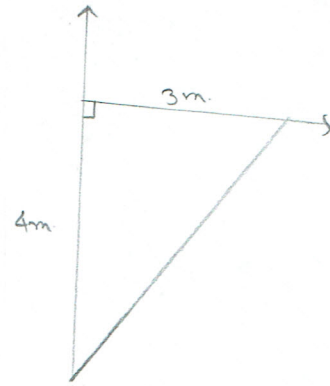
$$= 313.72 \text{ kN.}$$

Let 'h' be the location of pressure force then;

$$h = \bar{x} + \frac{I_{CG}}{A \cdot \bar{x}}$$

$$= 5.33 + \frac{3 \times 4^3}{36 \times 6 \times 5.33}$$

$$= 5.4967 \text{ m.}$$



d).

$$\text{Area of the gate (A)} = \frac{1}{2} \times 3 \times 4 = 6 \text{ m}^2.$$

Let 'h' be the vertical height of the triangle such that;

$$\frac{1}{2} \times 3 \times h = 6.$$

$$\therefore h = 2.4 \text{ m.}$$

$$\bar{x} = 4 + \frac{2}{3} \times 2.4$$

$$= 5.6 \text{ m.}$$

$$\therefore F = \gamma A \bar{x}$$

$$= 9810 \times 6 \times 5.6$$

$$= 329616 \text{ N.}$$

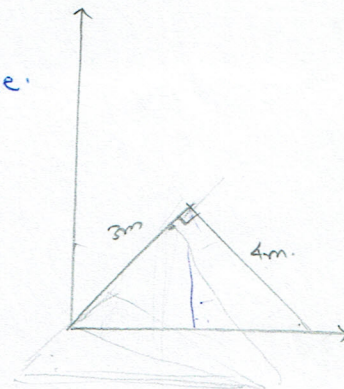
$$= 329.616 \text{ kN.}$$

If 'y' be the location of pressure force;

$$y = \bar{x} + \frac{I_{CG}}{A \bar{x}}$$

$$= 5.6 + \frac{5 \times 2.4^3}{36 \times 5.6 \times 6}$$

$$= 5.657 \text{ m.}$$



311bharati

8. You are in a car that falls into a lake to a depth as shown below. What is the moment about the hinges of the car door. (1.0 x 1.2 m) due to hydrostatic pressure? Can you open the door? What should you do?

Solution:

Given;

$$\text{Area (A)} = 1 \times 1.2 \text{ m} = 1.2 \text{ m}^2.$$

Now,

Pressure force on the door = P .

$$\therefore P = \gamma A \bar{x}$$

$$= 9.81 \times 1 \times 1.2 \times 8.6.$$

$$= 101.2 \text{ kN}.$$

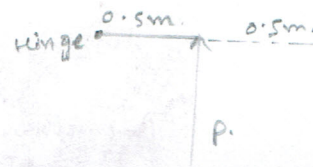
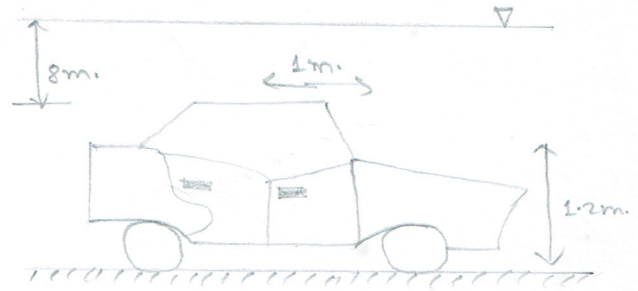
Then,

the moment about the hinge of the door is;

$$M = 101.2 \times 0.5.$$

$$= 50.6 \text{ kNm}.$$

So, the door can be opened when this moment is overcome.



Alibhaz

8. Gate AB in the figure shown is 1.2 wide and is hinged at A. Gauge G reads -0.147 bar and oil of $s=0.75$ is in the right hand side of the tank. What horizontal force must be applied at B for equilibrium of gate AB?

Solution;

$$\text{Air Pressure } (P_{\text{air}}) = -0.147 \times 1.01 \times 10^5 \text{ Pa} \\ = -14847 \text{ Pa}$$

$$P_A = P_{\text{air}} + \gamma_{\text{H}_2\text{O}} (5.5 - 1.8) \\ = -14847 + 9810 (5.5 - 1.8) \\ = -14847 + 36297 \\ = 21450 \text{ Pa}$$

P_1 on gate AB is given by;

$$P_1 = 1.8 \times P_A \times 1.2 + \frac{1}{2} \times (\gamma_{\text{H}_2\text{O}} \times 1.8) \times 1.8 \times 1.2 \\ = 46332 + 19070.64 \\ = 65402.64 \text{ N } (\rightarrow)$$

P_2 on gate AB due to oil is given by;

$$P_2 = \gamma_{\text{oil}} \times \bar{x}_{AB} \times A \\ = 0.75 \times 9810 \times 0.9 \times 1.8 \times 1.2 \\ = 14302.98 \text{ N } (\leftarrow)$$

Net pressure force acting towards right $= P_1 - P_2$.

$$= 65402.64 - 14302.98 \\ = 51099.66 \text{ N}$$

For the depth at which force P_2 is acting;

$$h_2 = \bar{x} + \frac{I_{C_2} \sin^2 \theta}{A \bar{x}} = 0.9 + \frac{1.2 \times 1.8^3}{12 \times 1.2 \times 1.8 \times 0.9} = 1.2$$

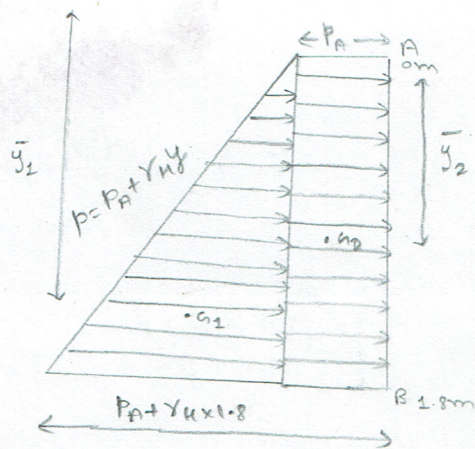
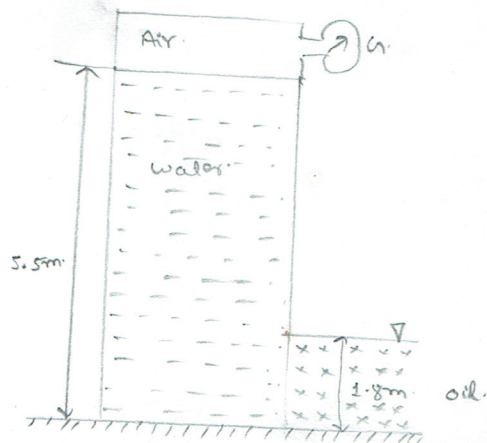
For the depth at which P_1 acts on the gate (h_1). Distance of combined centroid of the pressure diagram above;

$$\text{for } h_1; \bar{y}_1 = \frac{2}{3} \times 1.8 = 1.2 \text{ m}$$

$$\text{for } h_2; \bar{y}_2 = \frac{1}{2} \times 1.8 = 0.9 \text{ m}$$

$$\bar{y} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2}{A_1 + A_2} = \frac{38610 \times 0.9 + 15892.2 \times 1.2}{38610 + 15892.2} = 0.98 \text{ m}$$

$$\therefore h_1 = 0.98 \text{ m}$$



311bhan

Let F be the required force to be applied at 'B'. Then $\sum M_A = 0$. Since 'A' is a hinge.

$$\text{or, } -P_1 \times 0.98 + P_2 \times 1.2 + F \times 1.8 = 0$$

$$\text{or, } \frac{64094.5872 - 17163.576}{1.8} = F$$

$$\therefore F = 26072.788 \text{ N}$$

$$= 26.072 \text{ kN}$$

10. Determine the moment at point (hinged) 'A' required holding the sluice gate as shown in figure. If there is water on the other side also upto a level 'A', determine the resultant force on the gate, including its line of action.

Solution;

Case I;

If $\bar{x}$ be the vertical depth of C.G from free surface.

$$\bar{x} = 90 + \frac{10}{2} = 95 \text{ m}$$

$$\text{Area of surface (A)} = 10 \times 10 = 100 \text{ m}^2$$

Then; $P = \rho A \bar{x}$

$$= 9810 \times 100 \times 95$$

$$= 93195000 \text{ N}$$

If 'h' be the position of central force then;

$$h = \bar{x} + \frac{I_{CG}}{A \bar{x}}$$

$$= 95 + \frac{10 \times 10^3}{12 \times 100 \times 95}$$

$$= 95.0877 \text{ m. of pressure}$$

Then, distance of centre from 'A'.

$$d = h - 90$$

$$= 95.0877 - 90 = 5.0877 \text{ m}$$

Let 'M' be the moment at 'A' then;

$$\sum M = 0$$

$$\text{or, } M - P \times d = 0$$

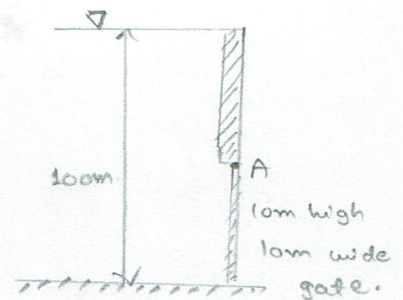
$$\text{or, } M = 93195000 \times 5.0877$$

$$\therefore M = 4.74 \times 10^8 \text{ Nm}$$

Again;

If 'P' be the pressure force on other side and $\bar{x}'$ be centre of gravity then;

$$\bar{x}' = 10/2 = 5 \text{ m}$$



311bha

$$P' = \gamma A \bar{x}'$$

$$= 9810 \times 100 \times 5$$

$$= 4905000 \text{ N.}$$

Location of Pressure force from free surface ;

$$d' = \bar{x}' + \frac{I_{\text{CG}}}{A \bar{x}'}$$

$$= 5 + \frac{10 \times 10^3}{12 \times 10^2 \times 5}$$

$$= 6.67 \text{ m.}$$

Thus, moment on 'A' is;

$$\sum M = 0.$$

$$\text{or, } M = P \times d + P' \times d' = 0.$$

$$\text{or, } M = 474150000 - 4905000 \times 6.67$$

$$= 441450 \text{ KNm.}$$

Again;

$$P_{\text{total}} = P - P'$$

$$= 93195000 - 4905000$$

$$= 88290 \text{ kN.}$$

If 'x' be the distance where the line of action of pressure force is

$$M = P_{\text{total}} \times x = 0.$$

$$\text{or, } 441450 = 88290 \times x.$$

$$\therefore x = 5 \text{ m.}$$

311bha

11. The figure below shows a gate of 4m width. Find the total force on the gate, the point of application of the force and the minimum force required to keep the gate closed.

Solution;

Given;

width of the gate = 4m.

Volume of the prism alongside;

$$= \frac{1}{2} \times 3 \times 3 \times 4 + 3 \times 4 \times 3$$

$$= 647460 \text{ N}$$

So pressure force on the gate is;

$$P = 647460 \text{ N}$$

$$= 647.460 \text{ kN}$$

Depth at which the pressure force acts;

$$\bar{h} = \bar{x} + \frac{I_{CG} \sin^2 \theta}{A \bar{x}}$$

$$= 5.5 + \frac{4 \times 3^3}{12 \times 3 \times 4 \times 5.5}$$

$$= 5.636 \text{ m}$$

Let 'x' be the moment arm then;

$$x = 7 - \bar{h}$$

$$= 7 - 5.636$$

$$= 1.364 \text{ m}$$

Moment about hinge due to pressure force P (M_1) = $P \times x$.

$$\therefore M_1 = 883.135 \text{ kNm}$$

For the gate to be closed an equivalent amount of anticlockwise moment is needed. Let the force for this balancing moment be F' and acts 'x' high from the hinge.

Then, for equilibrium;

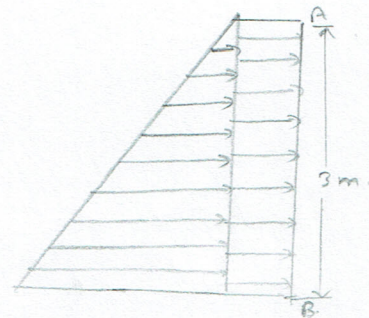
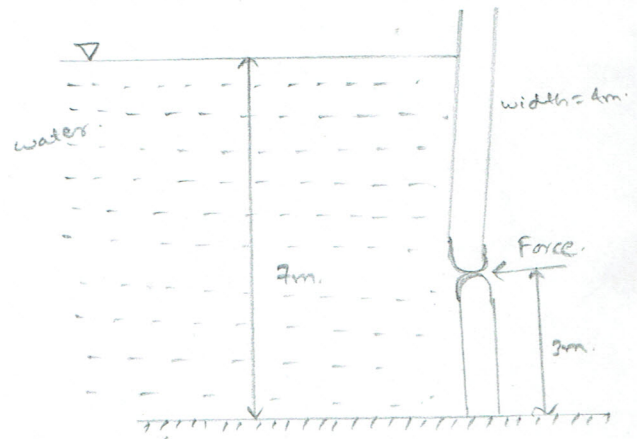
$$F \times x = P \times x = 883.135$$

The value of F depends upon the height of line of action;

$$F = \frac{883.135}{x} \text{ kNm}$$

For minimum force 'x' must be maximum i.e. $x = 3 \text{ m}$.

$$\therefore F_{\min} = 294.378 \text{ kN}$$



311bhai

12. Find the total resultant hydrostatic pressure force and its point of application on the dam as shown in the figure.

Solution;

where;

$$\bar{x} = 7.5 \text{ m.}$$

$$A = \frac{6 \times 15}{\sin 60^\circ} \quad [\because \text{width} = 6 \text{ m}]$$

Now;

$$P = \rho \bar{x} A$$

$$= 9810 \times 7.5 \times \frac{6 \times 15}{\sin 60^\circ}$$

$$= 7646138.29 \text{ N}$$

$$= 7646.138 \text{ kN}$$

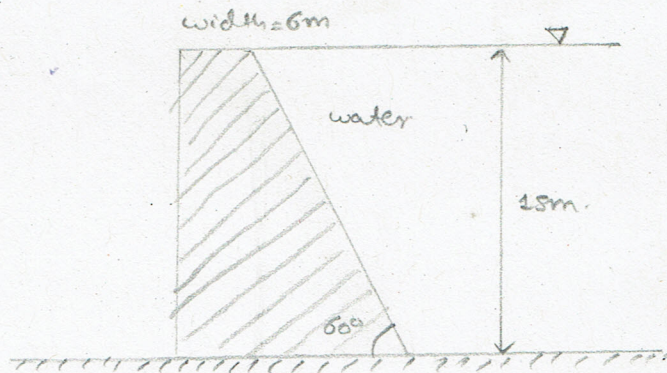
Let $\bar{h}$ be the location of pressure force. Then;

$$\bar{h} = \frac{\bar{x} + \frac{I_{CG} \sin^2 60^\circ}{A \bar{x}}}{\sin 60^\circ}$$

$$= \frac{7.5 + 6 \times \left(\frac{15}{\sin 60^\circ} \right)^3 \times \sin^2 60^\circ}{\frac{12 \times 15}{\sin 60^\circ} \times 6 \times 7.5}$$

$$= 10 \text{ m.}$$

$$\therefore \bar{h} = 10 \text{ m.}$$



311bhan

13.

calculate the magnitude of the resultant pressure force and its point of application on a dam as shown in the figure. make calculation.

Solution;

Given;

$$\text{width}(w) = 6\text{m.}$$

$$\bar{h} = \frac{18}{2} = 9\text{m.}$$

$$\begin{aligned}\text{Area of surface (A)} &= 18 \times 6 \\ &= 108\text{m}^2.\end{aligned}$$

Now; For horizontal component of pressure force.

$$\begin{aligned}P_x &= \gamma A \bar{h} \\ &= 9810 \times 108 \times 9 \\ &= 9535320\text{N.}\end{aligned}$$

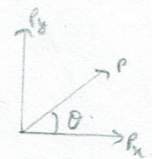
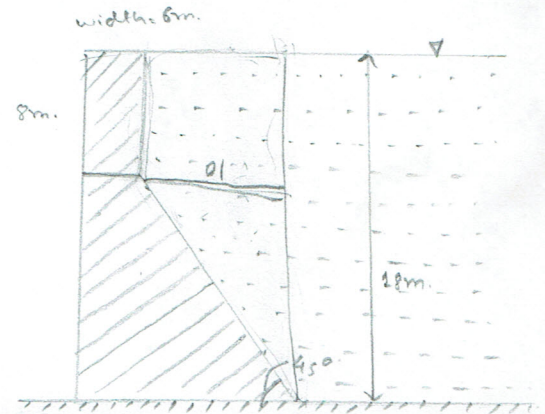
And; for vertical component of pressure force;

$$\begin{aligned}P_y &= \gamma V \\ &= 9810 \times \left(\frac{1}{2} \times 10 \times 10 + 8 \times 10 \right) \times 6 \\ &= 9810 \times (50 + 80) \times 6 \\ &= 7651800\text{N.}\end{aligned}$$

$$\begin{aligned}\text{Total force; } P &= \sqrt{P_x^2 + P_y^2} \\ &= \sqrt{9535320^2 + 7651800^2} \\ &= 12225889.76\text{N.} \\ &= 12225.88\text{KN.}\end{aligned}$$

And;

$$\begin{aligned}\theta &= \tan^{-1} \left(\frac{P_y}{P_x} \right) \\ &= \tan^{-1} \left(\frac{7651800}{9535320} \right) \\ &= 38.75^\circ.\end{aligned}$$



Alibhah

15. Find the minimum force required to keep the plate in place as shown in figure. Take width is unit.

Solution; For ^{total} horizontal force;

$$\bar{x} = \frac{6 \sin 35^\circ}{2}$$

$$= 1.72 \text{ m}$$

$$P = \gamma A \bar{x}$$

$$= 9810 \times 6 \times 1 \times 1.72$$

$$= 101282.12 \text{ N}$$

Let 'h' be the location of pressure force;

Then;

$$h = 1.72 + \frac{1 \times 6^3 \times \sin^2 35^\circ}{12 \times 6 \times 1.72}$$

$$= 1.72 + 1.744 \times \sin^2 35^\circ$$

$$= 2.30 \text{ m}$$

If 'l' be the inclined length at which pressure force acts then;

$$l = \frac{h}{\sin 35^\circ} = \frac{2.3}{\sin 35^\circ} = 4 \text{ m}$$

Now,

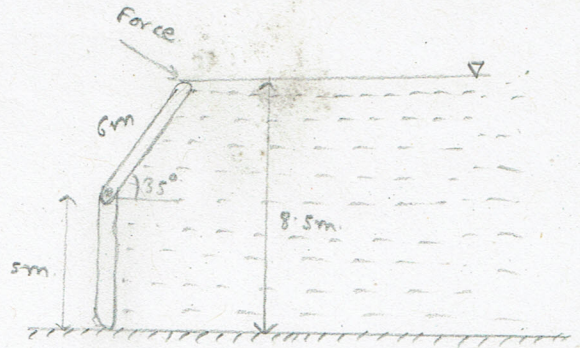
Taking moment about point B

$$F \times 6 = P \times (6 - 4)$$

$$\text{or } 3F = P$$

$$\text{or } 3F = 101282.12 \text{ N}$$

$$\therefore F = 33760.7 \text{ N}$$



311bha

26. A 1×1 m cover AB on a container under pressure as shown in figure is held in position by a force F. Calculate the force and the reaction at the hinge B.

Solution;

Given,

$$\text{Gauge Pressure} = 50 \text{ kN/m}^2$$

The position of a head at this condition is given as;

$$P = \gamma h$$

$$\therefore h = \frac{P}{\gamma}$$

$$= \frac{50 \times 10^3}{9810}$$

$$= 5.09 \text{ m}$$

$$= 5.09 \text{ m}$$

Now,

$$\bar{x} = 5.09 + \frac{1}{2} \times \sin 30^\circ$$

$$= 5.09 + 0.25$$

$$= 5.34 \text{ m}$$

$$\text{Pressure force (P)} = \gamma A \bar{x}$$

$$= 9810 \times 1 \times 1 \times 5.34$$

$$= 52452.50 \text{ N}$$

Let ' h_1 ' be the location of pressure force, then;

$$h_1 = \bar{x} + \frac{I_{CG} \sin^2 30^\circ}{A \bar{x}}$$

$$= 5.34 + \frac{1 \times 1^3}{12 \times 1 \times 5.35} \times \frac{1}{4}$$

$$= 5.34 + \frac{1}{256.80}$$

$$= 5.34 + 0.003894$$

$$= 5.3438 \text{ m}$$

If ' l ' be the inclined length from 'B' at which pressure force acts.

$$\therefore l = \frac{5.3438 - 5.09}{\sin 30^\circ} = 0.5077 \text{ m}$$

Taking moment about B

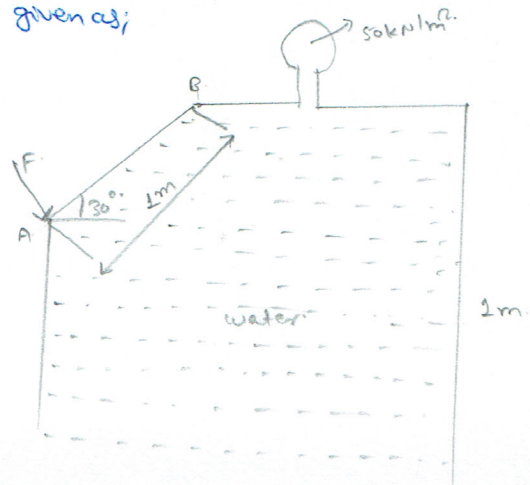
$$F \times l = 52452.50 \times 0.5077$$

$$\therefore F = 26630.13 \text{ N}$$

$$\text{Reaction force} = P - F$$

$$= 52452.50 - 26630.13$$

$$= 25822.36 \text{ N}$$



311bhan

17. Find the resultant pressure force due to static water on dams of 4m wide as shown in figure below.

For side A;

$$P_A = \gamma A \bar{x}_A$$

$$= 9810 \times 8 \times 4 \times 4$$

$$= 1255680 \text{ N}$$

$$\therefore P_A = 1255.68 \text{ kN} (\rightarrow)$$

$$h_A = \bar{x}_A + \frac{(I_{CA})_A \sin^2 90^\circ}{A \bar{x}_A}$$

$$= 4 + \frac{4 \times 8^3}{12} \times \frac{1}{8 \times 4 \times 4} = 5.333 \text{ m}$$

For side B;

$$P_B = \gamma A_B \bar{x}_B = 9810 \times \frac{5}{\sin 60} \times 4 \times 2.5$$

$$= 566380.6 \text{ N}$$

$$\therefore P_B = 566380.6 \text{ N}$$

$$h_B = \bar{x}_B + \frac{(I_{CB})_B \sin^2 60^\circ}{A_B \bar{x}_B}$$

$$= 2.5 + \frac{4 \times \left(\frac{5}{\sin 60}\right)^3}{12} \times \frac{\sin^2 60}{5/\sin 60 \times 4 \times 2.5}$$

$$= 3.35 \text{ m}$$

$$(P_B)_x = P_B \cos 60 = 283190 \text{ N}$$

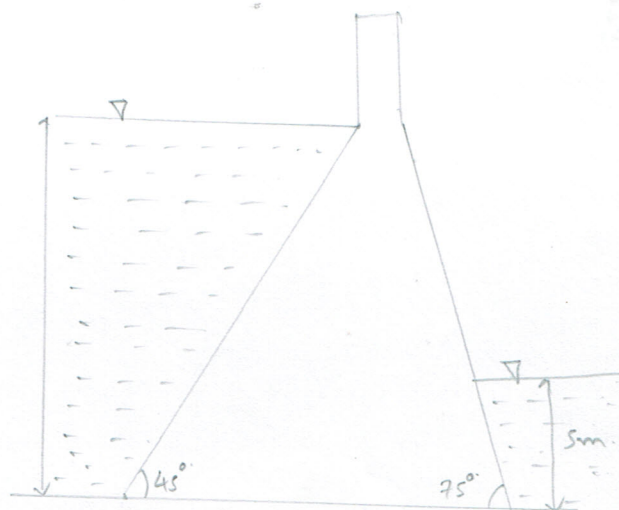
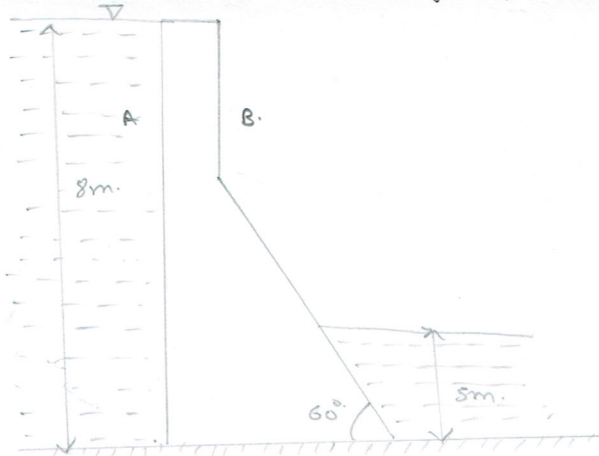
$$(P_B)_y = P_B \sin 60 = 490499 \text{ N}$$

$$P_x = 1255680 - 283190 = 972490 \text{ N}$$

$$P_y = 490499 \text{ N}$$

$$\therefore P = \sqrt{P_x^2 + P_y^2} = \sqrt{972490^2 + 490499^2}$$

$$= 1089.185 \text{ kN}$$



Bilbhan

For side A;

$$(P_x)_A = \gamma A \bar{x} = 9810 \times 50 \times 4 \times 25 = 49050 \text{ kN.}$$

$$(P_y)_A = \gamma V = 49050 \text{ kN.}$$

For side B

$$(P_x)_B = \gamma A \bar{x} = 9810 \times 3 \times 4 \times 2.5 = 490.5 \text{ kN.}$$

$$(P_y)_B = 9810 \times \frac{1}{2} \times 1.34 \times 5 = 32863.5 \text{ kN.}$$

Now;

Net horizontal force

$$\begin{aligned} P_x &= (P_x)_A - (P_x)_B = 4950 - 490.50 \\ &= 48559.5 \text{ N.} \end{aligned}$$

Net vertical force;

$$\begin{aligned} P_y &= (P_y)_A - (P_y)_B \\ &= 49050 - 32863.5 \\ &= 4917.137 \text{ kN.} \end{aligned}$$

$$\begin{aligned} \therefore \text{Total net force } (P) &= \sqrt{P_x^2 + P_y^2} \\ &= \sqrt{48559.5^2 + 4917.137^2} \\ &= 68997.8 \text{ kN.} \end{aligned}$$

$\therefore$ Hence, the net resultant pressure force is 68997.8 kN.

311bhar

19. Find the force P needed to hold the 10m long cylindrical object in position as shown in the figure below.

Solution;

For horizontal force on ABC

$$\bar{x} = 2\text{m}$$

$$A = 10 \times 4\text{m}^2$$

$$\therefore P_{x1} = \gamma \bar{x} A$$

$$= 9810 \times 2 \times 40$$

$$= 784.80\text{ kN} (\rightarrow)$$

The depth (location) of P_{x1}

$$h = \bar{x} + \frac{I_{G1} \sin^2 90^\circ}{A \bar{x}}$$

$$h = 2 + \frac{10 \times 4^3}{12 \times 2 \times 40}$$

$$\therefore h = 2.67\text{m}$$

For horizontal force on ADC;

$$\bar{x} = 1\text{m}$$

$$A = 10 \times 2\text{m}^2$$

$$\therefore P_{x2} = \gamma \bar{x} A$$

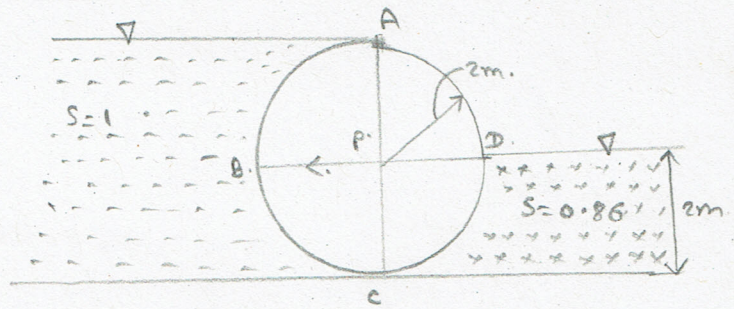
$$= 1 \times 10 \times 9810 \times 0.86$$

$$= 168.732\text{ kN} (\leftarrow)$$

$\therefore$ Force required to hold the object $= P_{x1} - P_{x2}$

$$= 784.80\text{ kN} - 168.732\text{ kN}$$

$$= 615.26\text{ kN}$$



311bhan

20. What force P is needed to hold the 4m wide gate as shown in the figure below

Solution;

For horizontal force,

$$P_h = \gamma \bar{x}_y A_y$$

$$= \gamma \times \left(6 + 3 \times \frac{1}{2}\right) \times (8 \times 4)$$

$$= 9810 \times \left(\frac{21}{2}\right) \times 12$$

$$= 882.3 \text{ kN}$$

For horizontal force,

$$P_y = \gamma V$$

$$= \gamma \times \left(6 \times 3 + \frac{\pi \times 3^2}{4}\right) \times h$$

$$= 987.69 \text{ kN}$$

$\therefore$ wetted part is outside the included area.

So, P_y acts upward.

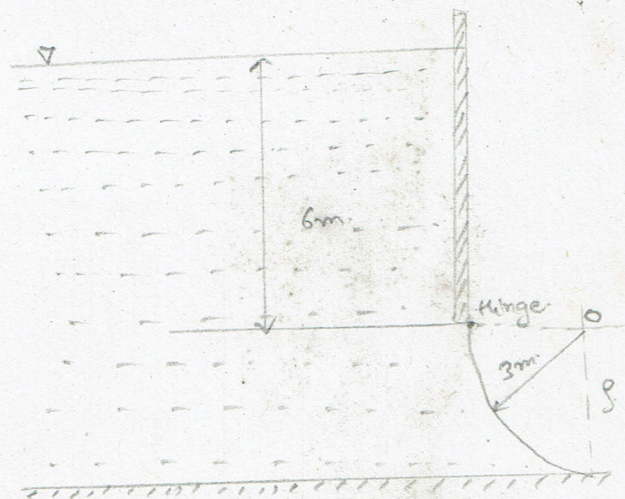
$$\text{Depth of } P_h (h) = \frac{\bar{x}_y + \frac{I_{xy} \sin 2\theta}{A_y \bar{x}_y}}{1}$$

$$= 7.5 + \frac{4 \times 3^3}{12 \times 4 \times 3 \times 7.5}$$

$$= 7.60 \text{ m}$$

$$d = \frac{4 \times 3}{3\pi} \text{ left of } O$$

$$= 1.27 \text{ m left of } O$$



311bhar

21. A sluice gate consists of a quadrant of a circle of radius 1.5m pivoted at its centre O. When the water is level with the gate, calculate the magnitude and direction of the resultant hydrostatic force on the gate and the moment required to open the gate. The width of the gate is 3m and it has a mass of 6 tonnes.

Solution;

Given, radius = 1.5m.

width = 3m.

Mass = 6 tonnes.

For, Horizontal force;

$$P_x = \gamma \bar{x} \bar{y} A_y$$

$$= 9810 \times \frac{1.5}{2} \times 1.5 \times 3$$

$$= 33108 \text{ N. } (\leftarrow)$$

For vertical force;

$$P_y = \gamma V$$

$$= 9810 \times \frac{\pi \cdot 1.5^2}{4} \times 3$$

$$= 52028.03 \text{ N. } (\uparrow)$$

$$\text{Depth of } P_x \text{ (h)} = \bar{y} + \frac{I_{xy} \sin 90^\circ}{A_y \bar{x}}$$

$$= \frac{1.5}{2} + \frac{3 \times 1.5^3}{12 \times 1.5 \times 3 \times \frac{1.5}{2}}$$

$$= 1\text{m.}$$

$$d = \frac{4 \times 1.5}{3\pi} = 0.637\text{m.}$$

$$\text{Resultant force (P)} = \sqrt{P_x^2 + P_y^2} = 61.64 \text{ kN.}$$

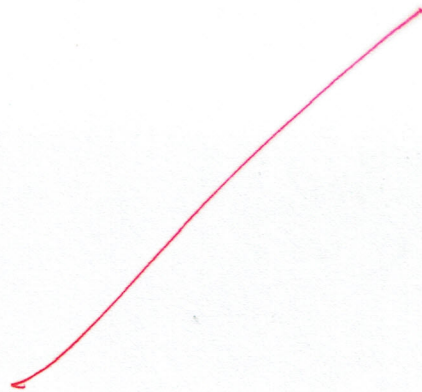
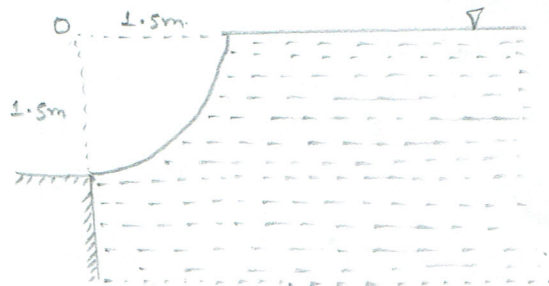
If 'θ' be the angle 'P' makes with the horizontal then;

$$\theta = \tan^{-1} \frac{P_y}{P_x} = 57.51^\circ$$

Total moment about O (M₀) is wxd - P_yxd + h₁ + h₂

$$= 37.477 \text{ kNm. } (+\curvearrowright)$$

So, anticlockwise moment is needed to open the gate.



311bha

23. The face of a dam is curved according to the relation $y = \frac{x^2}{2.4}$, where y and x are in meters, as shown in the diagram. Calculate the resultant force on each member run of the dam. Determine the position at which the line of action of the resultant force passes through the bottom of the dam.

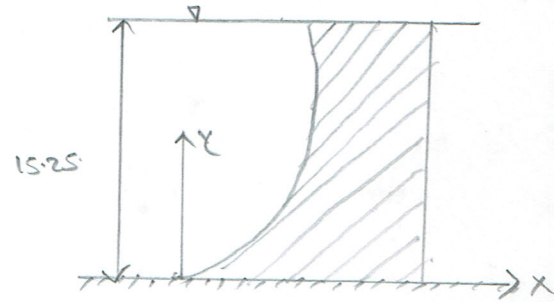
Solution;

Here;

Horizontal component of pressure force is;

$$P_x = 9810 \times 15.25 \times \frac{15.25}{2} \times 10^{-3} \text{ kN}$$

$$= 1141 \text{ kN}$$



$$P_y = \gamma V = 9810 \times \int_0^{15.25} x dy = 9810 \times 1.55 \times \frac{(15.25)^{3/2}}{3/2} \times 10^{-3} \text{ kN}$$

$$= 603.69 \text{ kN}$$

So total pressure force is;

$$P = \sqrt{P_x^2 + P_y^2} = \sqrt{1141^2 + 603.69^2} = 1290.86 \text{ kN}$$

And;

$$\tan \theta = \frac{P_y}{P_x} \Rightarrow \theta = \tan^{-1} \frac{603.69}{1141} = 27.88^\circ$$

Again; P_x acts at depth.

$$h = 7.625 + 2.545$$

$$= 5.08$$

$$\therefore \bar{y} = 5.08 \text{ m}$$

$$\text{So, } AC = \sqrt{15.25 \times 2.4} = 6.05 \text{ m}$$

P_y acts at the centroid.

$$\therefore \bar{x} = \frac{1}{A} \int x dA = \frac{\left[\frac{x^4}{4} \right]_0^{6.05}}{\left[\frac{x^3}{3} \right]_0^{6.05}} = 4.54 \text{ m}$$

So, pressure force passes through (4.54, 5.08 m).

$$\text{Actual position} = 4.54 + \frac{5.08}{\tan 27.88} = 14.142 \text{ m}$$

311bhan

24. Find the force P required holding the gate in the position as shown in the figure below. The gate is 5m wide.

Solution;

Given;

$$\text{width} = 5\text{m.}$$

$$\text{radius}(r) = 2\text{m.}$$

For horizontal force;

$$P_x = \gamma \bar{x}_y A_y$$

$$= \gamma \times 2 \times 5$$

$$= 9810 \times 10$$

$$= 98100 \text{ N. } (\rightarrow)$$

For vertical force;

$$P_y = \gamma V$$

$$= \gamma \times \text{Area of OAB} \times 5$$

$$= \gamma \times 5 \times \frac{\pi \times 2^2}{4}$$

$$= 154157.14 \text{ N } (\downarrow)$$

Let 'h' be the depth of pressure force.

$$\therefore h = \frac{\gamma \bar{x}_y + \frac{I \sin^2 \theta}{A_y \bar{x}_y}}{\gamma}$$

$$= \frac{1 + \frac{5 \times 2^3}{12 \times 5 \times 2 + 1}}{1}$$

$$= 1.33 \text{ m}$$

$$d = \frac{4 \times 2}{3\pi} = 0.848 \text{ m}$$

$$\text{moment arm of a force } P = 2 + 0.8 = 2.8 \text{ m}$$

$$\text{moment arm of force } P_x = 2 - h = 2 - 1.333 = 0.6667 \text{ m}$$

$$\text{moment arm of force } P_y = 0.848 \text{ m}$$

For equilibrium;

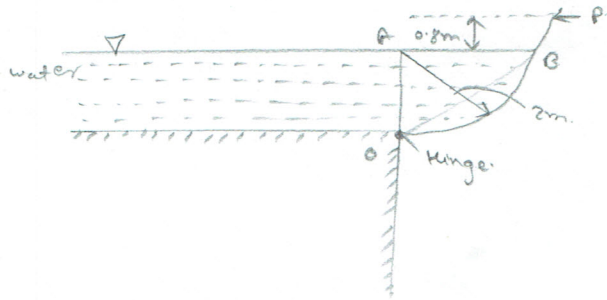
$$\sum M_o = 0$$

$$P_x \times \bar{x}_x + P_y \times \bar{y}_y - P \times 2.8 = 0$$

$$\therefore 98100 \times 0.6667 + 154157.14 \times 0.848 - P \times 2.8 = 0$$

$$\therefore P = 70045.90 \text{ N}$$

$$= 70.04 \text{ kN}$$



311bha

25. The 3m wide circular gate as shown in figure below weighs 400N with CG 0.9m to the left of the hinge. Estimate the force 'P' needed to open the gate.

Solution;

$$P_H = \gamma \bar{x} \bar{y} A_y$$

$$= \gamma \times (10 + 1.5) \times (3 \times 3)$$

$$= 1015.335 \text{ kN} (\rightarrow)$$

Line of action of P_H .

$$h = \bar{x}_y + \frac{I_{xx} \sin^2 \theta}{A_y \bar{x}_y}$$

$$= \left(10 + \frac{3}{2}\right) \times \frac{3 \times 3^3}{12 \times 3 \times 3 \times (10 + 1.5)}$$

$$= 11.58 \text{ m}$$

$$P_V = \gamma V$$

$$= \gamma \times \text{Area of ABKF} / F \times 3$$

$$= \gamma \times \left((10 \times 3) + \frac{3 \times 3^2}{4} \right) \times 3$$

$$= 1091 \text{ kN} (\uparrow)$$

$$d = \frac{1.5 \times 10 \times 3 + \frac{4 \times 3}{32} \times \frac{3 \times 3^2}{4}}{10 \times 3 + \frac{3 \times 3^2}{4}}$$

$$= \frac{45 + 9}{30 + 2.25}$$

$$= 1.45 \text{ m}$$

Here; For the gate to open;

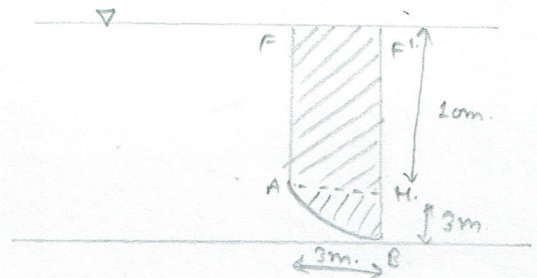
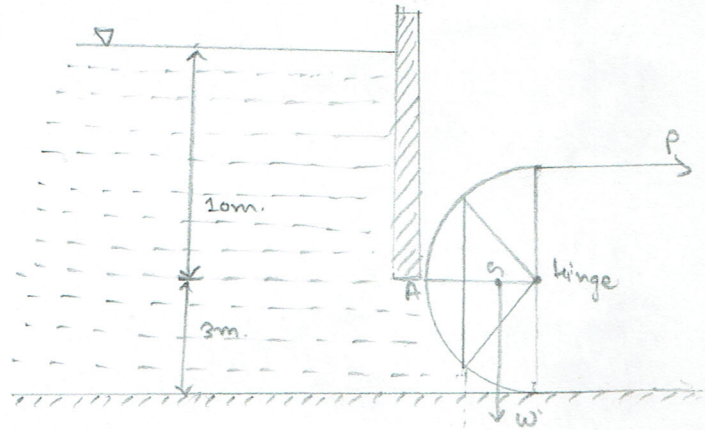
$$P_H \times (h - 10) + w \times 0.9 = P_V \times 1.45 + 3P$$

$$\text{or, } 1604.23 + 0.36 = 1581.95 + 3P$$

$$\therefore P = 7.546 \text{ kN}$$

when the weight $w = 400 \text{ kN}$;

$$P = 127.42 \text{ kN}$$



311bhan

26. Find the resultant pressure force of the liquid on the surface ABCDEF of 10m width as shown in the figure below ($S=1.2$).

Solution;

Given;

$$\text{width} = 10\text{m.}$$

$$S = 1.2.$$

$$A_y = 10.25 \times 10.$$

Now;

$$P_n = \gamma \times A_y \times \bar{x}_y.$$

$$= 1.2 \times 9.810 \times 10.25 \times 10 \times (2 + 5.125).$$

$$= 8597.23 \text{ kN.}$$

For vertical force;

$$P_y = \gamma V_{ABCA} + \gamma V_{D'EFD} - \gamma V_{C'DD'C'} \quad (\uparrow)$$

$$= \gamma \left[\left(\frac{1}{2} \times 2 \times 2 + \frac{1}{2} \times 2 \times 2 \right) \times 10 + \frac{7 \cdot 1.25^2}{2} \times 10 - (2 \times 4) \times 10 \right].$$

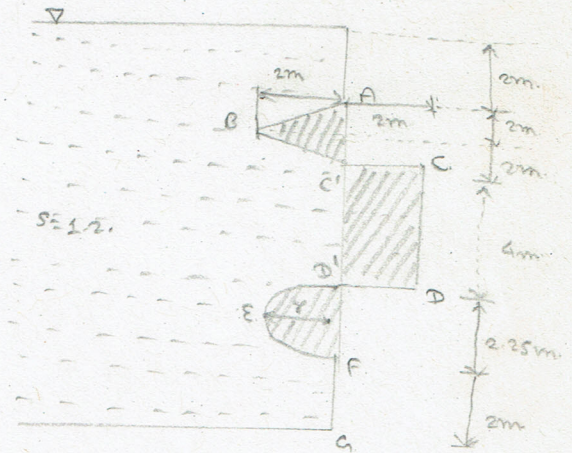
$$= -236.84 \text{ kN}$$

$$\therefore P_y = 236.84 \text{ kN} (\downarrow).$$

$$\therefore P = \sqrt{P_n^2 + P_y^2}$$

$$= \sqrt{8597.23^2 + 236.84^2}$$

$$= 8600.49 \text{ kN.}$$



311bhan

27. If the weightless quarter cylinder gate is in static equilibrium as shown in figure below, what is the ratio between specific weights?

Solution;

Given;

radius (r) = 4m.

- For liquid γ_1 .

For horizontal component of pressure force;

$$\bar{x} = 2.$$

$$\text{Area} = 4 \times \text{breadth} = 4 \times b.$$

$$\gamma = \gamma_{H_2O} \times \gamma_1.$$

$$\therefore P_x = \gamma A \bar{x}$$

$$= 9810 \times \gamma_1 \times 8b$$

$$= 78480b\gamma_1 \text{ --- a.}$$

For vertical component of pressure force;

$$\text{Volume (V)} = \frac{\pi r^2}{4} \times \text{breadth.}$$

$$= \frac{\pi \times 4^2}{4} \times b.$$

$$= 4\pi b.$$

$$= \frac{4 \times 22}{7} \times b.$$

$$= 12.571b.$$

$$\therefore P_y = 9810 \times \gamma_1 \times 12.571b.$$

$$= 123325.7143b\gamma_1$$

$$\theta = \tan^{-1} \left(\frac{P_y}{P_x} \right).$$

$$\text{or } \theta = \tan^{-1} (1.5714)$$

$$\therefore \theta = 57.52^\circ.$$

now;

In $\triangle ABC$.

$$\sin 57.51 = \frac{AB}{BC}$$

$$\therefore AB = 3.37m.$$

$$\cos 57.51 = \frac{AC}{BC}$$

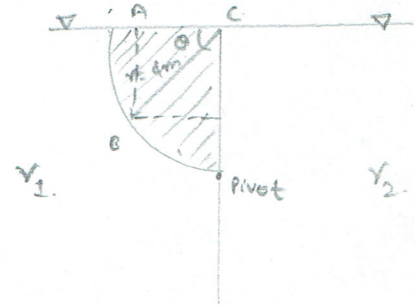
$$\therefore AC = 2.14m.$$

- For liquid γ_2 .

$$P_x' = \gamma_2 \times 9810 \times 4 \times b \times 2.$$

$$= \gamma_2 \times 9810 \times 8b.$$

$$= 78480b\gamma_2.$$



311bha

Let 'h' be the location of pressure force. Then;

$$\frac{h \bar{x} + \frac{I_{CG}}{A \bar{x}}}{}$$

$$= \frac{2 + 4^3 \times 2.14}{12 + 4 \times 2.14 \times 2}$$

$$= 2.67 \text{ m}$$

Now; Taking moment about 'Pivot'

$$P_x \times (4 - 3.37) + P_y \times 2.14 = P_x (4 - 2.67)$$

$$\text{or, } 784806 \gamma_1 \times 0.63 + 123325.71436 \gamma_1 \times 2.14 = 784806 \gamma_2 \times 1.33$$

$$\text{or, } 313359.4286 + 263917.1429 \gamma_1 = 104378.40 \gamma_2$$

$$\therefore \frac{\gamma_1}{\gamma_2} = 0.333$$

$$\therefore \frac{\gamma_2}{\gamma_1} = 3$$

311bhan

28. The cylinder is 2.4m long and is pivoted at O. Calculate the moment about O, required to hold it in position.

Solution;
Given;
 $S = 1$.

length = 2.4m.

For horizontal force,

$$P_x = \gamma \bar{x}_y A_y$$

$$= \gamma \times \frac{5}{2} \times 2.4 \times 5$$

$$= 294300 \text{ N}$$

$$= 294.30 \text{ kN}$$

For vertical forces,

$$P_y = \gamma V$$

$$= \gamma \times \frac{\pi \times 2.5^2}{2} \times 2.4$$

$$= 231235.714 \text{ N}$$

$$= 231.23 \text{ kN}$$

Position of P_x ;

$$h = \bar{x}_y + \frac{I_{yy} \sin^2 \theta}{A_y \bar{x}_y}$$

$$= \frac{2.5 + 2.4 \times 5^3}{12 \times 2.4 \times 5 \times 2.5}$$

$$= 3.33 \text{ m}$$

Position of P_y ;

$$d = \frac{4 \times 2.5}{3\pi}$$

$$= 1.06 \text{ m}$$

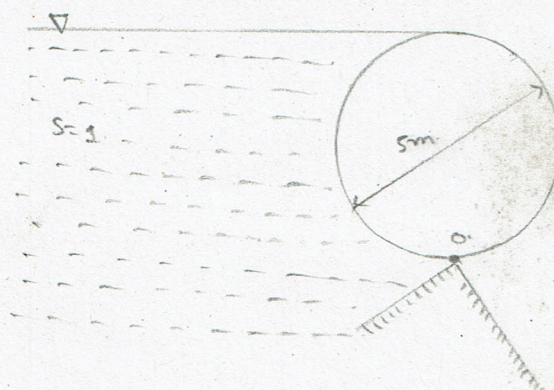
$$\text{Total moment about O (M}_O\text{)} = P_y \times 1.06 + P_x \times (5 - h)$$

$$= 231.14 \times 1.06 + 294.3 \times 1.667$$

$$= 735.51 \text{ kNm (}\downarrow\text{)}$$

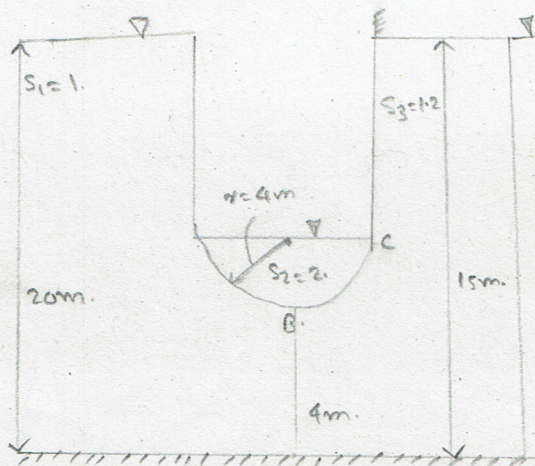
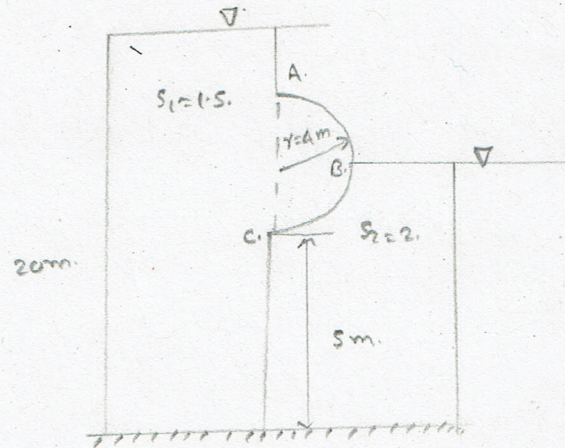
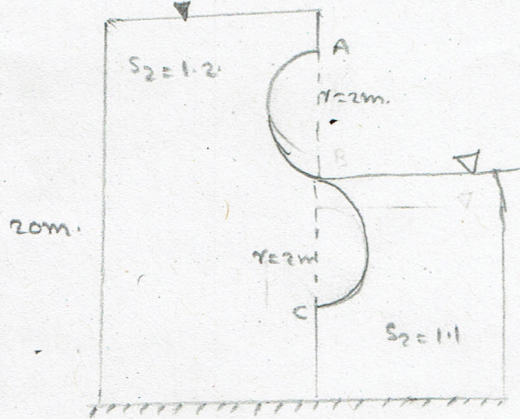
$\therefore$ An anticlockwise moment is required to maintain the equilibrium

311bhan



- 29) Find the resultant horizontal and vertical components of pressure forces on a curve surface ABC of 100m length as shown in figure below.

Solution;



For curve AB.

$$P_H = \gamma_2 A \bar{x}$$

$$= 1.28 \times 4 \times 100 \times (8 + 2)$$

$$= 47088 \text{ kN } (\rightarrow)$$

$$P_V = \gamma_2 V(A, B)$$

$$= 1.28 \times \frac{\pi \times 2^2}{2} \times 100$$

$$= 7336.56 \text{ kN } (\uparrow)$$

For curve BC.

$$P_H = \gamma_2 A \bar{x}_2 - \gamma_1 A \bar{x}_1$$

$$= 1.28 \times 100 \times 4 \times 14 - 1.18 \times 4 \times 100 \times 2$$

$$= 57290.4 \text{ kN } (\rightarrow)$$

$$P_V = \gamma_2 V(1) - \gamma_1 V(2)$$

$$= 1.28 \times 100 \times \frac{\pi \times 2^2}{2} - 1.18 \times 100 \times \frac{\pi \times 2^2}{2}$$

$$= 616.38 \text{ kN } (\downarrow)$$

311bhar

$$\therefore \text{Total vertical force on ABC} = 6780.18 (\uparrow) \text{ kN}$$

$$\begin{aligned} \text{Total horizontal force on Surface ABC} &= 47088 + 57290.4 \text{ kN} \\ &= 104378.4 (\rightarrow) \text{ kN} \end{aligned}$$

$$b) P_{x1} = \gamma_1 A_{AC} \bar{x}_1 \text{ (on ABC)}$$

$$\begin{aligned} &= 1.58 \times 8 \times 100 \times (7+4) \\ &= 129492 \text{ kN } (\rightarrow) \end{aligned}$$

$$P_{x2} = \gamma_2 A_{BC} \bar{x}_2 \text{ (on BC)}$$

$$\begin{aligned} &= 28 \times 4 \times 100 \times 4_{1/2} \\ &= 156960 \text{ kN } (\leftarrow) \end{aligned}$$

$$\therefore \text{Net } P_x = P_{x1} - P_{x2} = 113736 \text{ kN } (\rightarrow)$$

$$P_{y1} = \gamma_1 V(A, C) \text{ on (ABC)}$$

$$= 1.58 + \frac{2 \times 4^2}{2} \times 100$$

$$= 36982.82 \text{ kN } (\downarrow)$$

$$P_{y2} = \gamma_2 V(B, C) \text{ on BC}$$

$$= 28 \times \frac{2 \times 4^2}{4} \times 100$$

$$= 24655.22 \text{ kN } (\uparrow)$$

$$\begin{aligned} \therefore \text{Net } P_y &= 36982.82 - 24655.22 \\ &= 12327.6 \text{ kN } (\downarrow) \end{aligned}$$

$$c) P_{x1} = \gamma A \bar{x}_1 \text{ (on AB)}$$

$$\begin{aligned} &= 8 \times 4 \times 100 \times (12+2) \\ &= 54936 \text{ kN } (\rightarrow) \end{aligned}$$

$$P_{x2} = \gamma_2 A \bar{x}_2 \text{ (on AB)}$$

$$= 28 \times 4 \times 100 \times 2$$

$$= 156960 \text{ kN } (\leftarrow)$$

$$P_{x3} = \gamma_3 A \bar{x}_3 \text{ (on BC)}$$

$$= 128 \times 4 \times 100 \times (7+2)$$

$$= 42379.2 \text{ kN } (\leftarrow)$$

$$P_{y1} = \gamma_1 V(A, B)$$

$$= 8 \times (4 \times 12 + \frac{2 \times 4^2}{2}) \times 100 = 59415.62 \text{ kN } (\uparrow)$$

$$P_{y2} = \gamma_2 V(A, C)$$

$$= 49310.44 \text{ kN } (\downarrow)$$

$$P_{y3} = \gamma_3 V(B, C)$$

$$= 37754.73 \text{ kN } (\uparrow)$$

$$\therefore P_x = 3139.2 \text{ kN } (\leftarrow)$$

$$P_y = 57859.9 \text{ kN } (\uparrow)$$

311bhan

30. A circular segment water gate 5m long, is set as shown in figure. water is stored up to the upper face of the water gate. obtain the magnitudes of the horizontal and vertical components of force and also the magnitude and the direction of the resultant force acting on this water gate.

Solution;

Given;

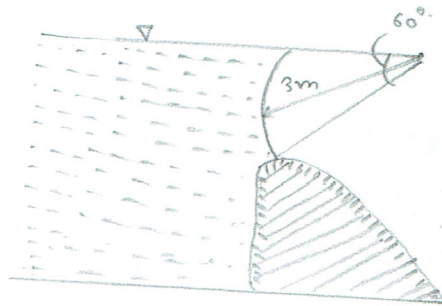
$$\text{radius}(r) = 3\text{m.}$$

$$\text{Height}(h) = 3 \times \sin 60^\circ.$$

$$= 3 \times \frac{\sqrt{3}}{2}.$$

$$= 2.60\text{m.}$$

$$\therefore \bar{x} = \frac{h}{2} = \frac{2.6}{2} = 1.3\text{m.}$$



Horizontal component of pressure force $(P_x) = \gamma A \bar{x}$

$$= 9810 \times 2.6 \times 5 \times 1.3.$$

$$= 165789\text{N.}$$

Vertical component of pressure force $(P_y) = \gamma V$

$$= 9810 \times [\text{Area of sector AOB} - \text{A of } \triangle AOC].$$

$$= 9810 \times \left[\frac{\pi}{6} \times 3^2 - \frac{1}{2} \times 2.6 \times 3 \cos 60^\circ \right] \times 5$$

$$= 1.35 \times 10^5\text{N.}$$

So resultant force, $P = \sqrt{P_x^2 + P_y^2}$

$$= \sqrt{1.35^2 + 1.65^2} \times 10^5.$$

$$= 2.13 \times 10^5\text{N.}$$

Now,

If θ be the angle made by the resultant force with the horizontal then;

$$\tan \theta = \frac{P_y}{P_x}$$

$$\therefore \tan \theta = \frac{1.35}{1.65}$$

$$\therefore \theta = \tan^{-1}(0.8181)$$

$$\therefore = 39.28^\circ.$$

Bilbhan

31. The cross section at tanks full of water under pressure is as shown in figure below. Determine the magnitude of the resultant force acting on the quarter circle curved surface BC, if the width of the tank is 20m.

Solution;

let 'h' be the height of liquid column equivalent to pressure -98.1 kN/m^2 .

$$\therefore h = \frac{-98.1}{9.81} \\ = -10\text{m}$$

Now;

If $\bar{x}$ be the depth of C.G. from free surface.

$$\bar{x} = 6 + 2 = 8\text{m}$$

$$\text{Area} = 4 \times 20 = 80\text{m}^2$$

For horizontal component of force;

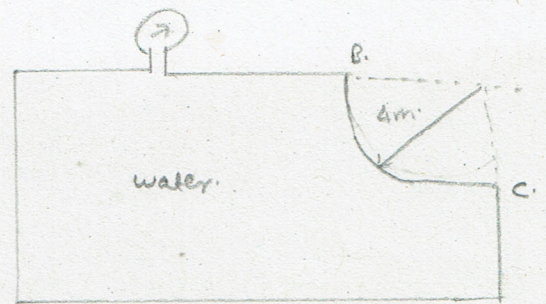
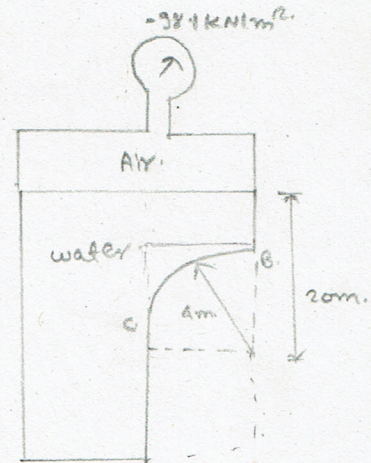
$$P_h = \gamma A \bar{x} \\ = 9810 \times 80 \times 8 \\ = 6278400 \text{ N } (\rightarrow)$$

For vertical component of force;

$$P_v = \gamma V \\ = 9810 \times \left[40 - \pi \times \frac{4^2}{4} \right] \times 20 \\ = 5382478.085 \text{ N}$$

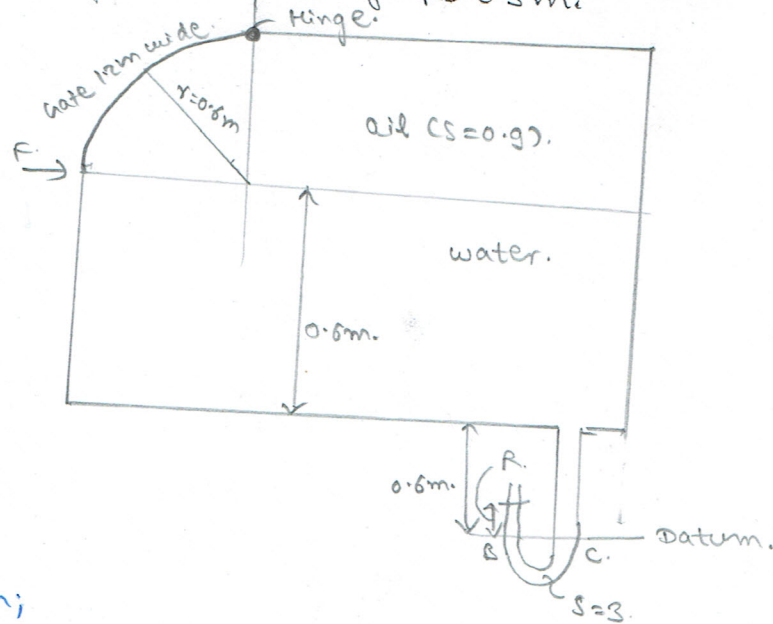
Then;

$$P = \sqrt{P_h^2 + P_v^2} \\ = \sqrt{6278400^2 + 5382478.085^2} \\ = 8.27 \times 10^6 \text{ N}$$



311bhan

32. What value of force F will just close the gate as shown in the figure below, if the value of $R = 0.5 \text{ m}$.



Solution;

$$P_B = 3 \times 9810 \times R$$

$$= 29430 \times 0.4$$

$$= 11772 \text{ Pa}$$

$$P_D = 11772 - 1.2 \times 9810$$

$$= 0 \text{ Pa}$$

i.e. Surface D is the free surface. So pressure above surface D is negative. Now;

$$P_x = \gamma A \bar{x} = 0.9 \times 9810 \times 0.6 \times 1.2 = 6356.88 \text{ N}$$

$$P_y = \gamma V = 9810 \times \frac{2 \times 0.6^2 \times 1.2}{4} = 3328.45 \text{ N}$$

The force P_x will act at depth.

$$h = \frac{0.6}{2} + \frac{1.2 \times 0.6^3}{12} \times \frac{1}{1.2 \times 0.6 \times \frac{0.6}{2}}$$

$$= 0.7 + 0.1$$

$$= 0.4 \text{ m from the surface D.}$$

i.e. at depth of 0.2 m from the hinge.

$$\text{The force } P_y \text{ acts at } \frac{4x}{3\pi} = \frac{4 \times 0.6}{3\pi} = 0.255 \text{ m left from hinge.}$$

Now, taking moment and hinge;

$$\sum M_{\text{hinge}} = 0$$

$$- 0.85 \times F + 0.255 \times 3328.45 + 0.2 \times 6356.88 = 0$$

$$0.85 F = 2120.12$$

$$\therefore F = 2494.27 \text{ N } (\leftarrow)$$

311bhar

1. Determine the density and volume of an object that weighs 3N in water and 4N in oil of specific gravity 0.83.

Solution;

we have;

wt. of body in liquid = wt. of body in air - upthrust due to liquid.

Given that, the weight of body in water = 3N.

wt. of body in water = wt. of body - upthrust.

$$\text{or wt. of body} = \text{wt. of body in water} + \text{upthrust} \\ = 3 + V_{\text{displace}} \times 9810 \quad \text{--- a).}$$

Similarly;

Given that, the wt. of body in oil = 4N.

wt. of body in liquid = wt. of body - upthrust.

$$\text{or wt. of body} = 4 + \text{upthrust} \\ = 4 + V_{\text{dis}} \times 0.83 \times 9810 \quad \text{--- b).}$$

From a) and b).

$$3 + V_{\text{dis}} \times 9810 = 4 + V_{\text{dis}} \times 0.83 \times 9810.$$

$$\text{or } V_{\text{dis}} \times 0.17 \times 9810 = 1.$$

$$\therefore V_{\text{dis}} = 5.966 \times 10^{-4} \text{ m}^3.$$

From a).

$$\text{wt. of body} = 3 + 5.966 \times 10^{-4} \times 9810.$$

$$\text{or, } 8 \times V \times g = 8.8823.$$

$$\text{or, } 8 \times 5.966 \times 10^{-4} \times 9.81 = 8.8823.$$

$$\therefore 8 = 1.51 \times 10^3 \text{ kg/m}^3.$$

Find the density of metallic body which floats at the interface of mercury and water such that 40% of its volume is submerged in mercury and 60% in water.

Solution;

Given; 40% of V is submerged in mercury.

60% of V is submerged in water.

now;

$$\text{wt. of body} = 0.6V \times \gamma + 0.4V \times \gamma \times 13.6.$$

$$\text{or, } V \times \gamma \times 8 = 0.6V \times \gamma + 0.4V \times \gamma \times 13.6.$$

$$\text{or } 8 = 0.6 + (0.4 \times 13.6)$$

$$\therefore 8 = 6.04.$$

$$\therefore \text{The density of object} = 6.04 \times 1000 \\ = 6040 \text{ kg/m}^3.$$

3116thar

3. How many weights of concrete of sp. wt. 25 kN/m^3 must be attached to a beam having volume of 0.1 m^3 and specific gravity 0.65 to cause both just sink in water?

Solution;

Given; sp. wt. $= 25 \text{ kN/m}^3$.

Volume $= 0.1 \text{ m}^3$.

Specific gravity $(G) = 0.65$.

Now;

$$\begin{aligned} \text{wt. of body} &= \text{wt. of concrete} + \text{wt. of beam.} \\ &= V_c \times \gamma_c + V_b \times \gamma_b \\ &= V_c \times 25 \times 10^3 + 0.1 \times 0.65 \times 9810 \\ &= 25 \times 10^3 V_c + 637.65 \text{ — (i)} \end{aligned}$$

$$\text{Upthrust} = (0.1 + V_c) \times 9810 \text{ — (ii)}$$

For the case of just sinking,

$$25 \times 10^3 V_c + 637.65 = 9810 + 9810 V_c$$

$$\text{or, } 15190 V_c = 9172.35 \quad 343.50$$

$$\therefore V_c = 0.02261 \text{ m}^3$$

$$\begin{aligned} \therefore \text{wt. of concrete} &= 25 \times 10^3 \times 0.02261 \\ &= 565.33 \text{ N} \end{aligned}$$

4. A cube 60 cm on an edge has its lower half of $S_1 = 1.4$ and upper half of $S_2 = 0.6$. It is submerged into two equally layered fluids, the lower of $S_3 = 1.2$ and upper $S_4 = 0.9$. Calculate the height of the top of cube above the interface.

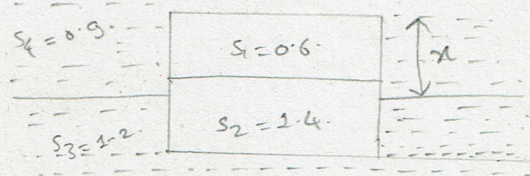
Solution;

Given; length of cube $= 60 \text{ cm} = 0.6 \text{ m}$.

$\therefore$ Volume of cube $= 0.6^3 = 0.216 \text{ m}^3$

$$\text{wt. of cube} = (9810 \times 1.4 + 9810 \times 0.6) \times \frac{0.216}{2}$$

$$= 2118.96 \text{ N}$$



If 'x' be the height of cube above the interface, then;

wt. of body = Upthrust (total)

$$2118.96 = x \times 0.9 \times 0.6^2 \times 9810 + (0.6 - x) \times 0.6^2 \times 9810 \times 1.2$$

$$\frac{9 \times 2118.96}{9810 \times 0.6^2} = 0.9x + (0.6 - x) \times 1.2$$

$$\text{or, } 0.60 = 0.9x + 0.72 - 1.2x$$

$$\therefore x = 0.40 \text{ m}$$

311bharat

5. An iceberg of specific gravity 0.92 is floating on a sea with a specific gravity of 1.025. If the volume of the iceberg above the water level is 100 m^3 , what is the volume of the iceberg?

Solution;

Given;

$$\text{Specific gravity (S)} = 0.92 \text{ (Iceberg)}$$

$$\text{Specific gravity of sea} = 1.025$$

We know;

$$\text{wt of iceberg} = \text{wt of sea water displaced}$$

$$\text{or, } V \times S_i \times \gamma_i = V_{\text{dis}} \times S_w \times \gamma_w$$

$$\text{or, } V \times 0.92 \times 9810 = V_{\text{dis}} \times 1.025 \times 9810$$

$$\therefore V_{\text{dis}} = 0.897V$$

Given that;

$$(V - V_{\text{dis}}) = 100$$

$$\text{or, } (V - 0.897V) = 100$$

$$\therefore V = 976.19 \text{ m}^3$$

6. A uniform wooden cylinder has relative density of 0.6. Determine the ratio of diameter to length so that it will just float upright in water.

Solution;

Given;

$$\text{Relative density} = 0.6$$

For the body to just float;

$$\text{wt of body} = \text{upthrust provided by liquid} \quad \text{--- a.}$$

Let the length of cylinder inside the water surface be 'x'.

$$\text{wt of wooden cylinder} = \frac{\pi D^2}{4} \times L \times 0.6 \times 9810 = 4624.71 D^2 L$$

$$\text{Volume displaced by cylinder} = \frac{\pi D^2}{4} \times x$$

Then; from a).

$$4624.71 D^2 L = \frac{\pi \times D^2 \times x \times 9810}{4}$$

$$\text{or, } 0.60 L = x$$

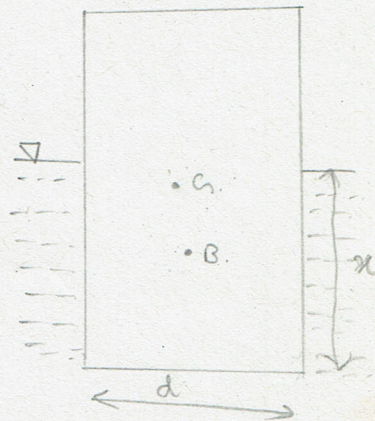
For the body to float in upright position.

$$\Sigma M = 0$$

$$\text{or, } B_m - B_G = 0$$

$$\text{For floating cylinder, } B_G = \frac{1}{2} (L - x)$$

$$= \frac{1}{2} (L - 0.60L) = 0.20L$$



311bha

$$\begin{aligned}
 \text{And; } Bm &= \frac{I_{CG}}{V_{\text{dis.}}} \\
 &= \frac{20 \text{ t.}}{\frac{20^2}{4} \times \pi \times 64} \\
 &= \frac{D^2}{10 \times 0.62} = \frac{D^2}{9.62}
 \end{aligned}$$

Now;

$$\frac{D^2}{9.62} = 0.22 \quad [\because Bm = BG].$$

$$\text{or, } \frac{D^2}{L^2} = 1.92.$$

$$\therefore \frac{D}{L} = 1.386.$$

7. A solid cylinder of length 150mm and diameter 75mm is to float upright in water. Between what limits its weight is?

Solution;

The body will be upright in water if;

1) the cylinder is just submerged in water;

wt. of cylinder = $\gamma_{\text{liquid}} \times V_{\text{dis.}}$

$$= 9810 \times \frac{22}{7} \times (0.075)^2 \times 0.15.$$

$$= 6.50 \text{ N.}$$

10. $Bm = BG.$

Let 'x' be the length of cylinder within water.

Also, $Bm = \frac{I_{CG}}{V_{\text{dis.}}}$

$$\begin{aligned}
 &= \frac{2 \times (0.075)^4}{64 \times 2 \times (0.075)^2 \times x} \\
 &= \frac{8}{25600x}.
 \end{aligned}$$

$$BG = \frac{1}{2} (L - x).$$

$$= \frac{1}{2} (0.150 - x).$$

Zillbhar

Also, wt. of cylinder = upthrust provided.

$$\text{or } \frac{\pi D^2}{4} \times L \times S \times 9810 = \frac{\pi D^2}{4} \times 9810 \times x.$$

$$\text{or } LS = x.$$

$$\therefore x = 0.155.$$

$$\therefore Bm > BG.$$

$$\frac{9.}{25600x} = \frac{1}{2} (0.150 - 0.1505).$$

$$\text{or } \frac{3 \times 2.}{1280} = 5 (0.150 - 0.1505).$$

$$\text{or } \frac{1.}{32} = 5 - 52.$$

$$\text{or } 325^2 - 328 + 1 = 0.$$

$$\therefore S_1 = 0.967; 0.032.$$

$$\therefore \text{wt. of cylinder} = \frac{\pi D^2}{4} \times L \times S \times 9810$$

$$= 6.287 \text{ N.}$$

8. A 25cm diameter cylinder is composed of material with 'S' = 0.8. will it float in water with the ends horizontal if its length is a) 30cm? b) 25cm? c) 20cm?

solution;

let 'x' be the part of cylinder inside the water.

Now;

wt. of cylinder = wt. of liquid displaced.

$$\text{or } \frac{\pi d^2}{4} \times L \times 0.8 = \frac{\pi d^2}{4} \times x.$$

$$\therefore x = 0.8 \times L \text{ --- a).}$$

$$\text{Again, } Bm = \frac{I_{CG}}{V_{dis}} = \frac{\pi \times 8^4}{4 \times \pi \times 2 \times x} = \frac{8^2}{4x} = \frac{d^2}{16x} = \frac{0.25^2}{16 \times 0.8L} = \frac{4.8828 \times 10^{-3}}{L}.$$

$$BG = \frac{L}{2} (1 - 0.8) = 0.1L.$$

$$Gm = Bm - BG = \frac{4.8828 \times 10^{-3}}{L} - 0.1L.$$

$$\text{for } L = 30\text{cm} = 0.3\text{m}; Gm = \frac{4.8828 \times 10^{-3}}{0.3} - 0.1 \times 0.3 = -1.2724 \times 10^{-2} < 0 \text{ (won't float).}$$

$$\text{for } L = 25\text{cm} = 0.25\text{m}; Gm = \frac{4.8828 \times 10^{-3}}{0.25} - 0.1 \times 0.25 = -5.46 \times 10^{-3} < 0 \text{ (won't float).}$$

$$\text{for } L = 20\text{cm} = 0.20\text{m}; Gm = \frac{4.8828 \times 10^{-3}}{0.20} - 0.1 \times 0.20 = 4.41 \times 10^{-3} > 0 \text{ (will float).}$$

Zilbhan

9. A cylindrical buoy 2m in diameter 2.5m high, weighs 18kN. Examine whether the buoy will or will not float with its axis vertical in sea water having specific gravity 1.025.

Solution;

wt. of cylindrical body $= 18 \times 10^3 \text{ N}$.

So,

$$\text{mass (m)} = \frac{18 \times 10^3}{9.81}$$

$$= 1834.86 \text{ kg}$$

$$\text{Volume (V)} = \pi r^2 l = \frac{\pi \times 4}{4} \times 2.5 = 7.85 \text{ m}^3$$

$$\text{Density } \rho = \frac{m}{V} = \frac{1834.86}{7.85} = 233.74 \text{ kg/m}^3$$

$$\therefore S_{\text{buoy}} = \frac{\rho_{\text{buoy}}}{\rho_{\text{water}}} = \frac{233.74}{1000} = 0.234$$

now;

$$W = F_b$$

$$\frac{\pi \times 4}{4} \times 2.5 \times 0.234 \gamma = \frac{\pi \times 4}{4} \times y \times \gamma \times 1.025$$

$$\Rightarrow y = 0.57 \text{ m}$$

And;

$$B.M. = \frac{I_{xx}}{V_{\text{dis}}} = \frac{\pi \times 4}{64} \times \frac{1}{\frac{\pi \times 4}{4} \times 0.57} = 0.438 \text{ m}$$

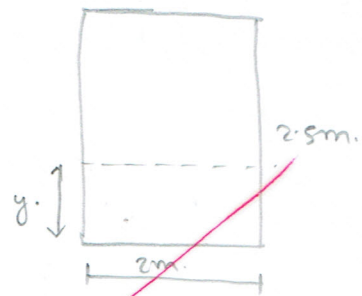
$$C.M. = B.M. - B_L$$

$$= 0.438 - (1.25 - 0.285)$$

$$= 0.438 - 0.965$$

$$\therefore C.M. = -0.527 \text{ m}$$

Since $C.M. < 0$ it is unstable.



311bhan

10. over what range of specific weights will a homogeneous cube float sides horizontal and vertical?

Solution;

'l': length of side of cube.

let 'x' be the depth at which the cube is immersed.

We know;

wt. of body = wt. of water displaced.

$$\text{or, } l^3 \times S \times \gamma = l^2 \times x \times \gamma.$$

$$\therefore x = Sl. \quad [S \text{ is the specific gravity of cube}]$$

For the body to float, relative specific gravity must be less than 1.

Now;

$$BM = \frac{I_{CG}}{V_{dis}} = \frac{l^4}{12 \times l^2 \times x} = \frac{l}{12S}.$$

$$BG = \frac{l}{2}(1-x) = \frac{l}{2}(1-S).$$

For the condition of a body to float

$$BM > BG.$$

$$\text{or, } \frac{l}{12S} > \frac{l}{2}(1-S).$$

$$\text{or, } 6S - 6S^2 < 1.$$

$$\text{or, } 6S^2 - 6S + 1 > 0.$$

$$\therefore S = 0.788$$

$\therefore S$ must be between 0.788 to 1. (i.e. of water).

311bhan

11. A cylinder of diameter 'd' made of material with specific gravity 'S' floats in liquid of specific gravity S_0 . Find the maximum length of the cylinder is to be stable with cylinder axis vertical $\left(\frac{d S_0}{\sqrt{8 S (S_0 - S)}} \right)$.

Solution;

Given;

Sp. gravity of liquid = S_0 .

Sp. gravity of cylinder = S .

Diameter of cylinder = d

Let 'L' be the length of the cylinder and 'x' be the dept upto which it is immersed. Then;

wt. of cylinder = wt. of liquid displaced.

$$\text{or, } \frac{\pi d^2}{4} \times L \times S = \frac{\pi d^2}{4} \times x \times S_0.$$

$$\therefore x = \frac{L S_0}{S}$$

we have;

$$B_m = \frac{I_{CG}}{V_{dis}} = \frac{\pi d^4}{64 \times \frac{\pi d^2}{4} \times x} = \frac{d^2}{16 x} = \frac{d^2 S_0}{16 L S}$$

$$B_G = \frac{1}{2} (L - x) = \frac{1}{2} \left(L - \frac{L S_0}{S} \right) = \frac{1}{2} L \left(\frac{S_0 - S}{S_0} \right)$$

Now;

$$G_M = B_M - B_G$$

$$= \frac{d^2 S_0}{16 L S} - \frac{1}{2} L \left(\frac{S_0 - S}{S_0} \right) = \frac{d^2 S_0^2 - 8 L^2 S (S_0 - S)}{16 L S S_0}$$

For stable condition;

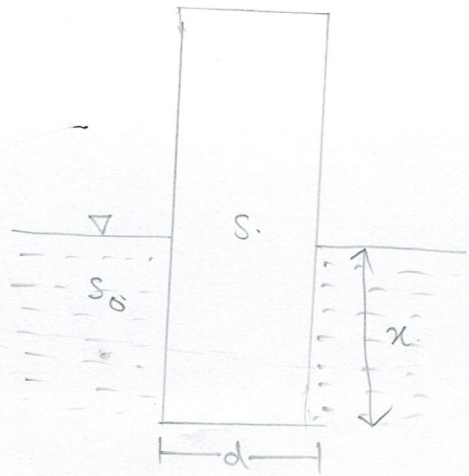
$$G_M \geq 0$$

$$\Rightarrow \frac{d^2 S_0^2 - 8 L^2 S (S_0 - S)}{16 L S S_0} \geq 0$$

$$\Rightarrow d^2 S_0^2 - 8 L^2 S (S_0 - S) \geq 0$$

$$\Rightarrow L^2 \leq \frac{d^2 S_0^2}{8 S (S_0 - S)}$$

$\therefore$ The maximum length possible is $L = \frac{d S_0}{\sqrt{8 S (S_0 - S)}}$ proved.



Zillbhar

12. A ship 12m wide and 30m long at the water line has a displacement of 6000kN. The centre of gravity is 40cm below the water line and the centre of buoyancy is 150cm further below it. Determine the metacentric heights for rolling and pitching axes of the ship.

Solution;

Given; wt. of ship (w) = $6000 \times 10^3 \text{ N}$.

width (b) = 12m.

length (L) = 30m.

we have;

wt. of ship = wt. of water displaced.

$$\text{or, } V_{\text{dis}} \times \rho \times g = 6000 \times 10^3$$

$$\text{or, } 12 \times 30 \times h \times 9810 = 6000 \times 10^3$$

$$\text{or, } 12 \times 30 \times 9810 \times h = 6000 \times 10^3$$

$$\therefore h = 1.7 \text{ m.}$$

$\therefore$ The height of ship below the water is 1.7m.

For rolling axes;

$$B_m = \frac{I_{CG}}{V_{\text{dis}}} = \frac{30 \times 12^3}{12 \times 12 \times 30 \times 1.7} = 7.058 \text{ m.}$$

$$B_G = 1.5 \text{ m.}$$

$$GM = B_m - B_G = 7.058 - 1.5 = 5.558 \text{ m.}$$

For pitching axes;

$$B_m' = \frac{I_{CG}'}{V_{\text{dis}}} = \frac{12 \times 30^3}{12 \times 12 \times 30 \times 1.7} = 44.11 \text{ m.}$$

$$\therefore GM' = B_m' - B_G$$

$$= 44.11 - 1.5$$

$$= 42.61 \text{ m.}$$

13. A cargo ship weighing 4000 tonnes has a draft of 7m in sea water ($\rho = 1.035$). After discharging cargo of 150 tonnes its draft reduces by 0.5m. what will be its draft in a fresh water harbor after further discharging a cargo by 300 tonnes.

Solution;

The ship has 7m draft when the weight is 4000 tonnes.

we know;

wt. of ship = wt. of water displaced.

$$\text{or, } V_{\text{dis}} \times \rho \times g = 4000 \times 9810$$

$$\text{or, } \text{Area} \times 7 \times \rho = 4000$$

$$\therefore \text{Area} = 552.1 \text{ m}^2$$

Bilbhad

When 450 tonnes is thrown, then let 'x' be the draft of the ship.

wt of ship = wt of liquid displaced

$$\text{or, } A \times x \times 9810 = (4000 - 450) \times 9810$$

$$\text{or, } 552.1 \times x = 3550$$

$$\therefore x = 6.43 \text{ m}$$

14. A block of ice floats in water. Find what fraction of its volume will be under water if $S = 0.90$.

Solution;

wt of ice = wt of liquid displaced.

$$\text{or, } V \times S \times 9810 = V_{\text{dis}} \times 9810$$

$$\text{or, } V \times 0.90 \times 9810 = V_{\text{dis}} \times 9810$$

$$\therefore V_{\text{dis}} = 0.90 \times \text{Volume of ice}$$

15. A body of volume 0.15 m^3 needed a force of 300N to keep it submerged in water. To keep it submerged in another liquid it needed a force of 200N. Find the specific gravity of the liquid.

Solution;

For the equilibrium of the forces, the sum of external force and weight of the body must be equal to buoyant force (F_b).

So, for water:

$$W + 300 = 9810 \times 0.15$$

$$\Rightarrow W = 1171.5 \text{ N}$$

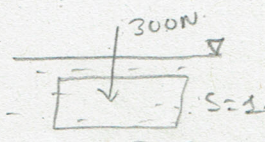
For other liquid:

$$W + 200 = \gamma_{\text{liquid}} \times 0.15$$

$$\Rightarrow \gamma_{\text{liquid}} = \frac{1371.5}{0.15}$$

$$\therefore \gamma_{\text{liquid}} = 9143.33 \text{ N/m}^3$$

$$\text{So, specific gravity of liquid } (S) = \frac{\gamma_L}{\gamma_{H_2O}} = \frac{9143.33}{9810} = 0.932$$



Bilbhan

18. A cylinder of diameter 400mm having specific gravity 0.9 floats in water. what is the maximum permissible length of the cylinder in order it may float in stable equilibrium it axis vertical?

Solution;

Given; diameter $(d) = 400\text{mm} = 0.4\text{m}$.

Specific gravity $(S) = 0.9$.

Let 'x' be the height of the cylinder immersed.

So; wt. of cylinder = wt. of liquid displaced.

$$\text{or, } \frac{\pi d^2}{4} \times l \times 0.9 \times g = \frac{\pi d^2}{4} \times x \times g$$

$$\therefore x = 0.9l.$$

$$\begin{aligned} B_m &= \frac{I_{\text{cyl}}}{V_{\text{dis}}} = \frac{\pi d^4}{64 \times \frac{\pi d^2}{4} \times x} \\ &= \frac{d^2}{16 \times 0.9l} \\ &= \frac{0.4^2}{16 \times 0.9l} \\ &= \frac{1}{90l} \end{aligned}$$

$$B_G = \frac{1}{2} [L - 0.9l]$$

$$= \frac{1}{20l}$$

For floating in stable equilibrium.

$$B_m = B_G$$

$$\text{or, } \frac{1}{90l} = \frac{1}{20l}$$

$$\text{or, } l^2 = \frac{2}{9}$$

$$\therefore l = 0.4714\text{m}$$

$$= 471.4\text{mm}$$

311bhan

17. A right circular cone with apex angle 60° of relative density 's' with respect to liquid in which it floats with apex angle downwards should float stable. Show that the stability is ensured for $0.42 < s < 1$.

Solution;

For floating, the 's' should be less than 1.

Now;

If H be the ht. of cone and 'x' be the depth with which the cone is immersed.

$$\frac{R}{H} = \frac{r}{x} = \tan 30^\circ \quad \text{--- (i)}$$

Now;

wt. of cone = wt. of liquid displaced.

$$\text{or } V \times s \times g = V_{\text{dis}} \times g.$$

$$\text{or } \frac{1}{3} \times \pi \times R^2 \times H \times s = \frac{1}{3} \times \pi \times r^2 \times x.$$

$$\text{or } x^3 \tan^2 30^\circ = H^3 \times s \times \tan^2 30^\circ.$$

$$\therefore x = H s^{1/3}$$

Now;

$$Bm = \frac{I_{CG}}{V_{\text{dis}}}$$

$$= \frac{\pi r^4}{4 \times \frac{1}{3} \times \pi r^2 \times x}$$

$$= \frac{3r^2}{4Hs^{1/3}} \quad \text{--- (a)}$$

Also;

$$Bg = \frac{3}{4} H - \frac{3}{4} x$$

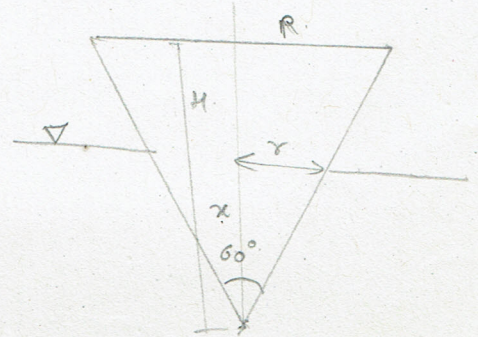
$$= \frac{3}{4} H (1 - s^{1/3}) \quad \text{--- (b)}$$

For floating; $Bm > Bg$.

$$\text{or } \frac{3}{4} \times \frac{x^2 \tan^2 30^\circ}{\pi} > \frac{3}{4} H (1 - s^{1/3}) \quad [\text{From (i)}]$$

$$\text{or } s^{1/3} > 3 - 3s^{1/3}$$

$$\text{or } 4s^{1/3} > 3$$



Bilbhan

$$\text{or } S > \left(\frac{3}{4}\right)^3.$$

$$\therefore S > 0.42.$$

So, the body can float on the condition of $0.42 < S < 1$.

19. A cone floats in water with vertex downwards. If the specific gravity of the cone material is 0.8, find the least apex angle of cone for stable equilibrium.

Solution;

$$B.C. = \frac{3}{4}l - \frac{3}{4}x = \frac{3}{4}(l-x).$$

$$W = F_b.$$

$$\text{or } 0.8 \times 9810 \times \frac{\pi d^2 l}{12} = 9810 \times \frac{\pi d'^2 x}{12}.$$

$$\text{or } 0.8 d^2 l = d'^2 x.$$

$$\text{or } \frac{d'^2}{d^2} \frac{x}{l} = 0.8.$$

Since, $\frac{d'}{x} = \frac{d}{l} \therefore \frac{d'}{d} = \frac{x}{l}.$

$$\text{or } \left(\frac{x}{l}\right)^3 = 0.8.$$

$$\therefore \frac{x}{l} = 0.9283. \Rightarrow x = 1.0772x.$$

Bm;

$$= \frac{I_{xx}}{V_{dis}} = \frac{\frac{\pi d^4}{64}}{\frac{\pi d'^2 x}{12}} = \frac{12}{64} \frac{d^2}{x}.$$

$$B.C. = B.m.$$

$$\text{or } \frac{12}{64} \times \frac{d^2}{x} = \frac{3}{4}(l-x)$$

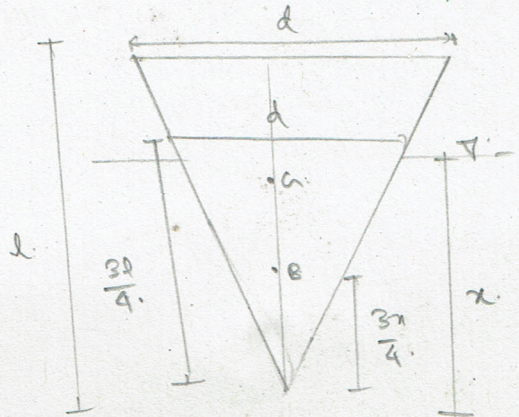
$$\text{or } d^2 = 4x(1.0772x).$$

$$\text{or } d' = 0.5558x.$$

$$\text{or } \frac{d'}{d} = 0.2779x.$$

$$\therefore \frac{d'}{d} = 0.2779. \Rightarrow \tan \theta = 0.2779 \therefore \theta = 15.5^\circ.$$

$$\therefore 2\theta = 31^\circ$$



311bhan

21. A hollow cylinder external radius R and internal radius r and height h with specific gravity S floats in a liquid of sp. gravity S_0 . So that for its stable equilibrium $h \leq S_0 \sqrt{\frac{R^2 + r^2}{2S(S_0 - S)}}$.

Solution;

Height of cylinder $= h$.

Sp. gravity of liquid $= S_0$.

Sp. gravity of cylinder $= S$.

Depth of immersion $= h'$.

wt. of cylinder $=$ wt. of displaced liquid.

Now;

$$\gamma_{\text{cylinder}} \cdot V_{\text{cylinder}} = \gamma_{\text{liq}} \cdot V_{\text{dis.}}$$

$$\Rightarrow S \times 9810 \times \pi \times (R^2 - r^2) \times h = 9810 \times S_0 \times \pi \times (R^2 - r^2) \times h'$$

$$\Rightarrow h' = \frac{Sh}{S_0} \quad \text{--- (i)}$$

Position of centre of buoyancy $(OB) = \frac{Sh'}{2S_0}$

Position of CG $(OC) = \frac{h}{2}$

$$BC = OC - OB = \frac{h}{2} - \frac{Sh'}{2S_0} = \frac{h}{2S_0} (S_0 - S)$$

$$MB = \frac{1}{V} \times \frac{\frac{1}{4} \pi \times (R^4 - r^4)}{\pi \times (R^2 - r^2) \times h'} = \frac{R^2 + r^2}{4 \frac{Sh'}{S_0}} \quad \therefore MB = \frac{S_0(R^2 + r^2)}{4Sh'}$$

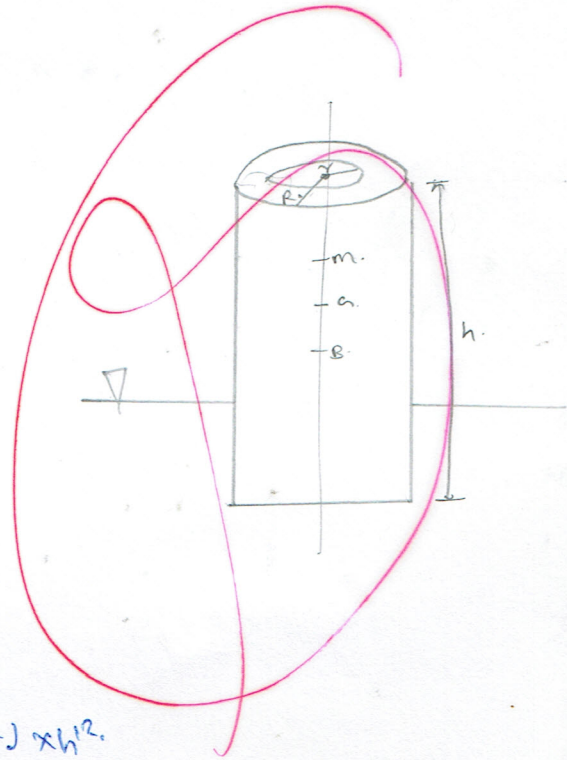
And;

$$GM = MB - BC = \frac{S_0(R^2 + r^2)}{4Sh} - \frac{h}{2S_0} (S_0 - S)$$

$$\text{So, } GM \geq 0.$$

$$\Rightarrow \frac{S_0(R^2 + r^2)}{4Sh} - \frac{h}{2S_0} (S_0 - S) \geq 0.$$

$$\Rightarrow h^2 \leq S_0^2 \frac{R^2 + r^2}{2^2 (S_0 - S)^2} \quad \therefore h \leq S_0 \sqrt{\frac{R^2 + r^2}{2S(S_0 - S)}} \quad \text{proved}$$



311bhan

Tutorial 4

Hydrostatics (Relative Equilibrium)

by
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1. An open rectangular container $2.5\text{m} \times 1\text{m}$ base and 2m height, half full with water is accelerated at 4m/sec^2 up on a 15° inclined along its length. Will the water spill? At what acceleration does the water begin to spill?

Solution:

Let θ be the angle the liquid makes with the free surface. Then;

$$\tan \theta = \frac{a \cos 15^\circ}{g + a \sin 15^\circ}$$

$$= \frac{3.863}{9.81 + 1.035}$$

$$= 0.35$$

$$\therefore \theta = \tan^{-1}(0.35)$$

$$= 19.60^\circ$$

Angle with the horizontal $= 19.60 + 15 = 34.60^\circ$.

For spilling of water;

$$\tan \theta = \frac{2}{2.5}$$

$$\therefore \theta = 38.65^\circ$$

$\therefore$ The water does not spill.

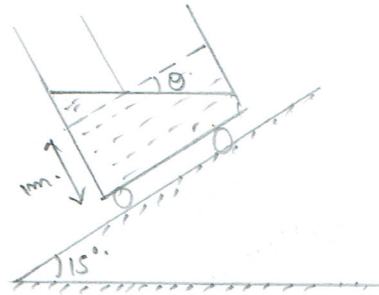
For the required acceleration (a').

$$\tan 38.65 = \frac{a' \cos 15^\circ}{g + a' \sin 15^\circ}$$

$$\therefore, 0.80 = \frac{a' (0.965)}{9.81 + a' (0.258)}$$

$$\therefore, 7.84 + a' (0.2070) = a' \times 0.965$$

$$\therefore a' = 10.34 \text{ m/s}^2$$



311bha

2. A circular cross-sectional tank of 2m depth and 1.3m diameter is filled with liquid and accelerated uniformly in horizontal direction. If $\frac{1}{3}$ of the liquid is spilled out, determine the acceleration.

Solution;

Total vol. of cylindrical tank;

$$V = \frac{2 \times 2 \times 1.3^2}{4} = 2.65 \text{ m}^3$$

$$\text{Spill volume} = \frac{1}{3} V = 0.885 \text{ m}^3$$

$$\therefore \text{Left volume} = 1.77 \text{ m}^3$$

Now;

$$\frac{2 \times 1.3^2}{4} \left(x + 1 - \frac{x}{2} \right) = 1.77$$

$$\text{or, } \frac{x}{2} = 0.334$$

$$\therefore x = 0.668 \text{ m.}$$

So,

$$\tan \theta = \frac{2 - 0.668}{1.3}$$

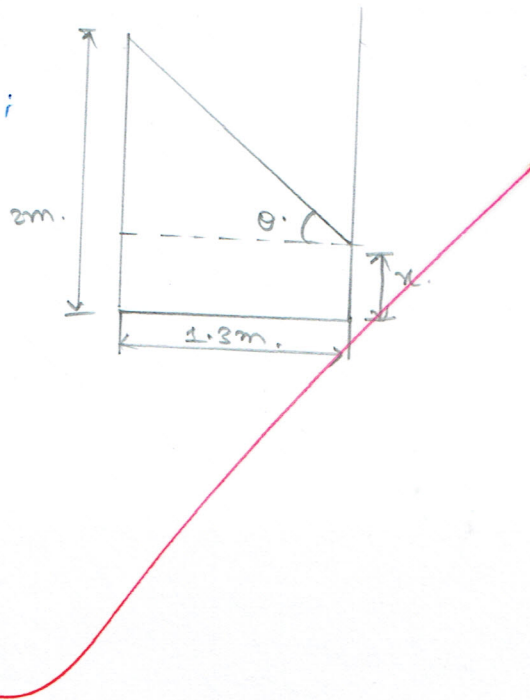
$$\therefore \tan \theta = 1.0246$$

Also we have;

$$\tan \theta = \frac{a_x}{g}$$

$$\text{or, } a_x = 9.81 \times 1.0246$$

$$= 10.05 \text{ m/s}^2$$



311bhan

3. An open rectangular tank 3.5m long 2m deep and 1.8m wide contains water to a depth 1.5m. Find the horizontal acceleration which may be imparted to the tank along its longer side so that;
- there is just no spilling of water from the tank.
 - the front bottom corner of the tank is just exposed.
 - the bottom of the tank is exposed to its mid-point.
- Proceed to calculate the total force on each end of tank.

Solution;

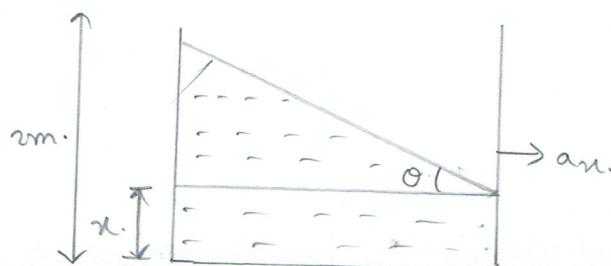
Given;

Length of the tank = 3.5m (L).

depth (H) = 2m.

Breadth (b) = 1.8m.

Depth of water (d) = 1.5m.



Let 'a' be the horizontal acceleration.

Then;

a. no spilling of water.

$$Z = 1.5 + 1.75 \tan \theta$$

$$\text{or, } 2 = 1.5 + 1.75 \times \tan \theta$$

$$\text{or, } 0.5 = 1.75 \tan \theta$$

$$\therefore \tan \theta = \frac{0.5}{1.75} = 0.28$$

$$\text{or, } \frac{a}{g} = 0.28$$

$$\therefore a = 2.80 \text{ m/s}^2$$

b. The front corner of the tank is just exposed.

$$\therefore \tan \theta = \frac{2}{3.5}$$

$$\text{or, } \tan \theta = 0.5714$$

$$\text{or, } \frac{a}{g} = 0.5714$$

$$\therefore a = 5.60 \text{ m/s}^2$$

$$\text{Pressure force (F)} = \gamma A \bar{h}$$

$$= 9810 \times 2 \times 1.8 \times 1$$

$$= 35.316 \text{ kN}$$

$$\text{Actual vol. of water (V)} = 3.5 \times 1.8 \times 1.5$$

$$= 9.45 \text{ m}^3$$

311bhan

$$\text{Volume of water left } (V_p) = \frac{1}{2} \times 2 \times 3.5 \times 1.8 \\ = 6.3 \text{ m}^3.$$

$$\therefore \text{Volume of water Spilled} = 9.45 - 6.3 \\ = 3.15 \text{ m}^3.$$

c) The bottom of the tank is exposed up to its mid-point.
 $h = 2 \text{ m}$ $b = 1.75 \text{ m}$.

we have;

$$\tan \theta = \frac{a}{g}$$

$$\text{or, } \frac{2}{1.75} = \frac{a}{g}$$

$$\therefore a = 11.21 \text{ m/s}^2$$

$$\text{Pressure force at an end } (P_1) = \gamma A \bar{x} \\ = 8810 \times 2 \times 1.8 \times \frac{2}{2} \\ = 35.316 \text{ kN}.$$

$$\text{Volume of water present } (V_p) = \frac{1}{2} \times 2 \times 1.75 \times 1.8 \\ = 3.15 \text{ m}^3$$

$$\therefore \text{Volume of water Spilled} = 9.45 - 3.15 \\ = 6.30 \text{ m}^3.$$

311bhan

4. A tube ABC bent at right angle open at A and closed at C, filled with water, is accelerated to the right as shown in figure below. Determine the acceleration at which the pressure at 'C' becomes atmospheric. would the pressure at B be different from what it is now?

solution;

we have;

$$\tan \theta = \frac{3}{4}$$

$$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\therefore \theta = 36.86^\circ$$

we have;

$$\tan \theta = \frac{a}{g}$$

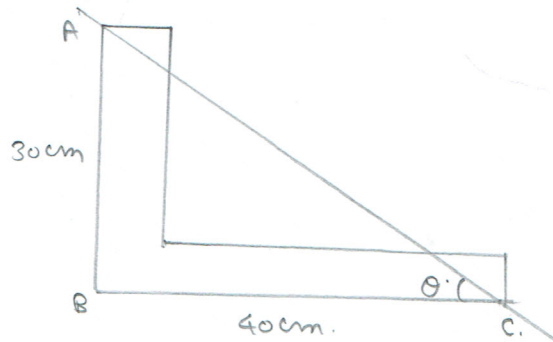
$$\therefore a = g \times \tan \theta$$

$$= 9.81 \times \frac{3}{4}$$

$$= 7.35 \text{ m/s}^2$$

$\therefore$ The required acceleration is 7.35 m/s^2 .

The pressure at 'B' would be same i.e. 9810×0.30
 $= 2943 \text{ Pa}$.



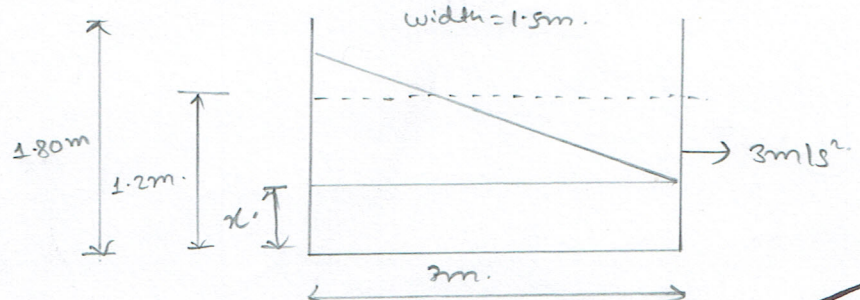
5. A rectangular tank 1.5 m wide 3 m long and 1.8 m deep contains water to a depth of 1.2 m. When it is accelerated horizontally at 3 m/s^2 in the direction of its length, calculate the depth of water at each end of the tank and the total force on each end of the tank. Check the difference between these forces by calculating the inertia of these accelerated mass.

solution;

$$\tan \theta = \frac{a_x}{g}$$

$$= \frac{3}{9.81}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{3}{9.81}\right) = 17^\circ$$



311bha

$$\tan 17^\circ = \frac{x_1}{3}$$

$$\Rightarrow x_1 = 0.917 \text{ m}$$

Here,

$$\text{Initial Volume} = 1.2 \times 3 \times 1.5 = 5.4 \text{ m}^3$$

Volume remains constant when the container moves with acceleration of 3 m/s^2 .

So,

$$5.4 = \frac{1}{2} \times 3 \times 0.917 \times 1.5 + x \times 3 \times 1.5$$

$$\Rightarrow x = \frac{3.34}{4.5} = 0.74 \text{ m}$$

$\therefore$ Height of water in rare end is $0.74 + 0.917 \text{ m} = 1.65 \text{ m}$.

Height of water in front end is 0.74 m .

Force in front end:

$$P_{\text{front}} = \gamma A_f \bar{x}_f = 9.81 \times 0.74 \times 1.5 \times \frac{0.74}{2} = 4.03 \text{ kN}$$

Force in rare end:

$$P_{\text{rare}} = \gamma A_r \bar{x}_r = 9.81 \times 1.65 \times 1.5 \times \frac{1.65}{2} = 20.25 \text{ kN}$$

So, net force is $P = 20.25 - 4.03 = 16.22 \text{ kN}$.

For, $a_x = 6 \text{ m/s}^2$

$$\tan \theta = \frac{6}{9.81}$$

$$\Rightarrow \theta = 31.45^\circ$$

Spill volume is,

$$V' = 5.4 - \frac{1}{2} \times 2.94 \times 1.8 \times 1.5$$

$$= 1.43 \text{ m}^3$$

Also;

$$\text{Net force} = ma$$

$$\text{or, } 16.22 = V \times 9 \times a$$

$$\text{or, } 16.22 = 1.5 \times 3 \times 1.2 \times 3$$

$$\text{or, } 16.22 \text{ kN} = 16.2 \text{ kN (True)}$$

Hence, verified.

Bibha

6. An open container of liquid accelerates up a 30° inclined plane at 5 m/sec^2 . What is the slope of the free surface? What are pressure intensities at the bottom of each end? What is the slope for the same acceleration in horizontal plane?

Solution;

Given; acceleration = 5 m/sec^2

Inclination $(\theta) = 30^\circ$.

we have;

$$\tan \theta = \frac{5 \cos 30^\circ}{9.81 + 5 \sin 30^\circ}$$

$$\text{or, } \tan \theta = \frac{4.33}{9.81 + 2.50}$$

$$\text{or, } \tan \theta = \frac{4.33}{12.31}$$

$$\therefore \theta = 19.37^\circ$$

$$\therefore \tan \theta = 0.35$$

For horizontal plane;

$$\tan \theta = \frac{a}{g}$$

Here, $a = 5 \text{ m/sec}^2$

$$\text{or, } \tan \theta = \frac{5}{9.81}$$

$$\therefore \tan \theta = 0.50$$

311bhan

7. A tank 18m long 5m wide and 4m deep is closed to the top and is completely filled; what will be the pressure difference between the left-hand end at the top and the right hand end at the top if the liquid has a specific wt. of 8 kN/m^3 and the horizontal acceleration is $0.3g$. Sketch planes of equal pressure, indicating their magnitude; assume zero pressure in the upper right corner.

Solution;

Given;

length of the tank (L) = 18m.

Breadth of the tank (b) = 5m.

Depth of the tank (h) = 4m.

acceleration (a) = $0.3g$.

Now;

$$\tan \theta = \frac{a}{g}$$

$$\therefore \frac{x}{18} = \frac{0.3g}{g}$$

$$\therefore x = 5.4\text{m.}$$

Pressure difference between left hand end at top and right hand end at top (P_1) = $\gamma_1 P$.

$$= 8 \times 5.4$$

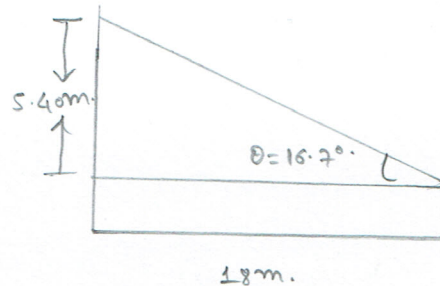
$$= 43.2 \text{ kPa.}$$

Pressure at bottom left (P_L) = $\gamma_B \times (4 + 5.4)$

$$= 8 \times 9.4 = 75.2 \text{ kPa.}$$

Pressure at bottom right (P_R) = $\gamma_B \times 4$.

$$= 32 \text{ kPa.}$$



3116ha24

8. A cylindrical vessel of diameter 200mm axis and height 0.3m is arranged to rotate about its vertical axis. The depth of water is 225mm in rest condition. Calculate the speed of rotation in radian per second at which a) the water will spill over the edge b). the axial depth is zero.

Solution;

Given;

$$\text{radius of vessel} = \frac{100}{2} \text{ mm} = 50 \times 10^{-3} \text{ m.}$$

$$\text{Depth of water (h)} = 0.225 \text{ m.}$$

Now;

- a) for the water to spill over the edge.

$$z = \frac{\omega^2 r^2}{2g}$$

$$\text{or } (\pi + y) = \frac{\omega^2 r^2}{2g}$$

$$\text{or } \omega^2 r^2 = 2x + 2g. \quad [\because \pi = g]$$

$$\text{or } \omega^2 r^2 = 2 \times 9.81 \times 2 \times (0.3 - 0.225)$$

$$\text{or } \omega^2 \times (50 \times 10^{-3})^2 = 2 \times 9.81 \times 2 \times 0.075$$

$$\therefore \omega = 34.32 \text{ rad/sec.}$$

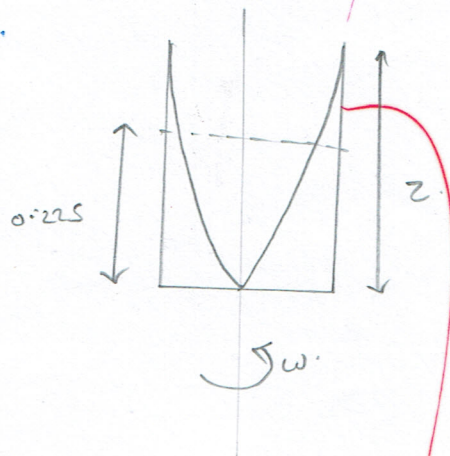
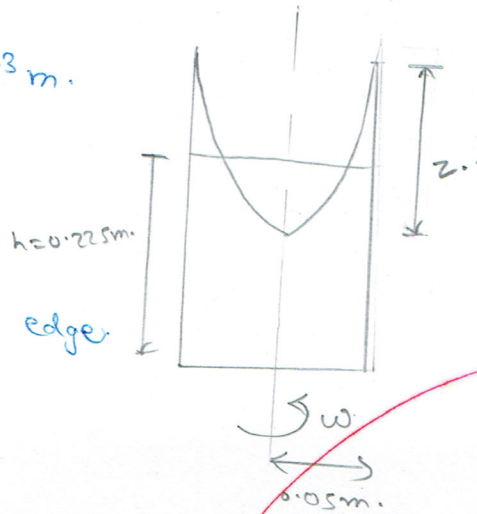
- b) for the axial depth to be zero.

$$z = \frac{\omega^2 r^2}{2g}$$

$$\text{or } 0.3 = \frac{\omega^2 r^2}{2 \times 9.81}$$

$$\text{or } \omega^2 \times (50 \times 10^{-3})^2 = 0.3 \times 2 \times 9.81$$

$$\therefore \omega = 48.52 \text{ rad/sec.}$$



311bhan

9. A cylindrical vessel 20cm in diameter and 30cm high contains oil of $S=0.8$ up to a height of 20cm. Calculate the constant rotational velocity about the vertical axis when the oil is at the verge of spilling over the brim.

Solution;

Let ω be the required angular velocity.

Now,

$$z = \frac{\omega^2 r^2}{2g}$$

$$\text{or } (x+y) = \frac{\omega^2 r^2}{2g}$$

$$\text{or } \omega^2 r^2 = 2g(x+y)$$

$$\therefore x=y$$

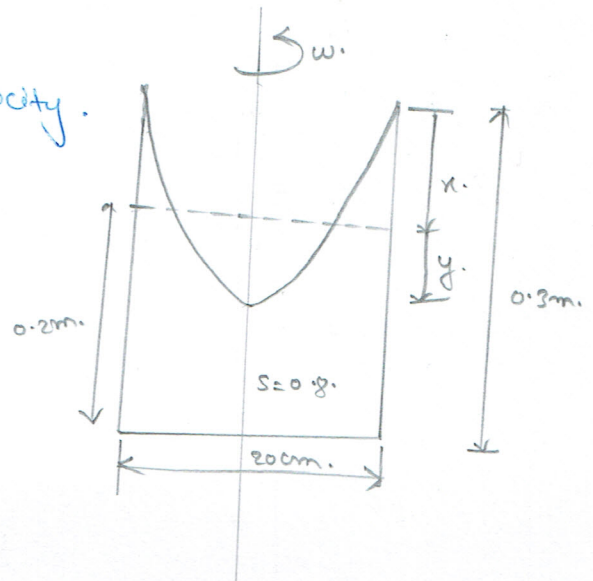
$$\text{or } \omega^2 r^2 = 2g \times 2x$$

$$\text{or } \omega^2 = \frac{4gx}{r^2}$$

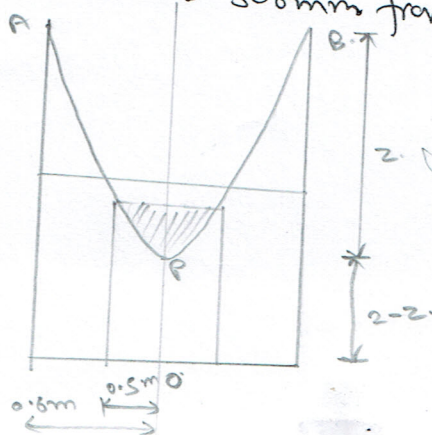
$$\text{or } \omega^2 = \frac{4 \times 9.81 \times 2 \times 0.1}{0.1^2}$$

$$\text{or } \omega^2 = 392.4$$

$$\therefore \omega = 19.8 \text{ rad/sec.}$$



10. A cylindrical vessel open at the top, 1200 mm diameter and 2000 mm high, is filled with water upto a depth of 1500 mm. The cylinder rotates about its axis at a constant speed of 90 rpm. Find the volume of water that spills out and the intensity of pressure developed at the bottom of the tank at the radius 500 mm from the axis.



311bha

Here;

$$z = \frac{\omega^2 R^2}{2g} = \frac{9.425^2 \times 0.6^2}{2 \times 9.81}$$

$$= 1.63 \text{ m}$$

$$\therefore \text{Height } P_0 = 2 - z = 2 - 1.63 = 0.37 \text{ m}$$

Now;

The volume of water that is spilled is;

$$V_{\text{spill}} = \text{Total initial volume} - (\text{Volume of liquid between the gap } P_0 + \text{Vol. of Paraboloid APB}).$$

$$= 1.5 \times \pi \times 0.6^2 - \left(0.37 \times \pi \times 0.6^2 + \frac{1}{2} \times \pi \times 0.6^2 \times 1.63 \right)$$

$$\therefore V_{\text{spill}} = 0.356 \text{ m}^3$$

Now, to find pressure intensity at radius of 500mm from 'O'.

$$\text{Head at the radius} = 0.37 + \frac{9.425^2 \times 0.5^2}{2 \times 9.81}$$

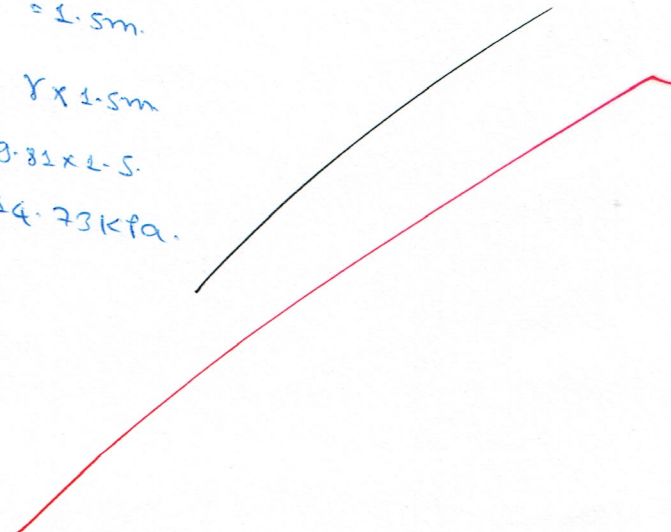
$$= 0.37 + 1.13$$

$$= 1.5 \text{ m}$$

$$\text{So pressure } (p) = \gamma \times 1.5 \text{ m}$$

$$= 9.81 \times 1.5$$

$$= 14.73 \text{ kPa}$$



311bhan

11. An upright U-tube manometer with its limbs 0.5m high and spaced apart contains a liquid to a height of 0.4m in each tube. If the U-tube is rotated at 10 rad/sec about a vertical axis at 0.1m from one limb, what would be the levels of the liquid be in the limbs?

Solution;

Here;

Total depth = 0.8m.

$$\begin{aligned} z_1 &= \frac{\omega^2 r_1^2}{2g} \\ &= \frac{10^2 \times 0.1^2}{2 \times 9.81} \\ &= 0.050 \text{ m.} \end{aligned}$$

$$\begin{aligned} z_2 &= \frac{\omega^2 r_2^2}{2g} \\ &= \frac{10^2 \times 0.2^2}{2 \times 9.81} \\ &= 0.2038 \text{ m.} \end{aligned}$$

$$\begin{aligned} z_1' &= z_1 + z_{\min} \\ &= 0.050 + z_{\min} \end{aligned}$$

$$z_2' = 0.20 + z_{\min}$$

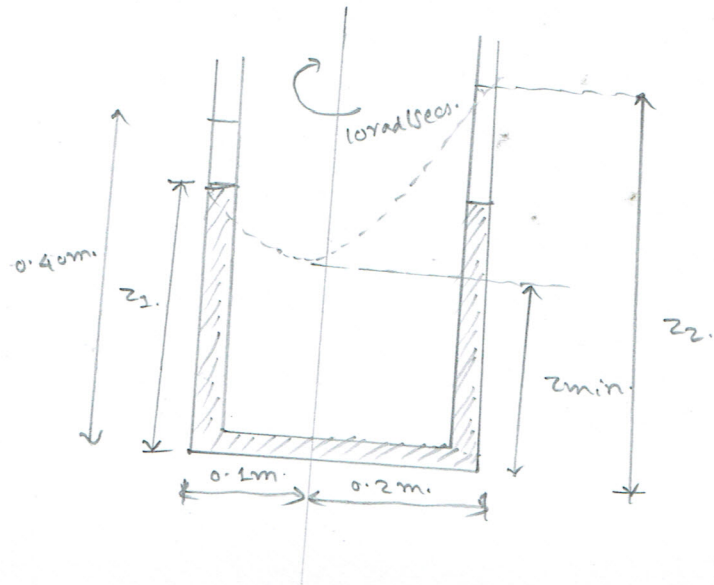
$$\therefore z_1' + z_2' = 0.25 + 2z_{\min}$$

$$\text{or, } 0.8 = 0.25 + 2z_{\min}$$

$$\therefore z_{\min} = 0.275 \text{ m.}$$

$$\therefore z_1' = 0.323 \text{ m.}$$

$$z_2' = 0.47 \text{ m.}$$



Zillbhar

12. A close vessel 120 cm in diameter is completely filled with oil of specific weight 8.3 kN/m^3 and rotated at 400 rpm . What will be the pressure difference between the circumference and axis of rotation?

Solution;

Given;

$$\gamma = 8.3 \times 10^3 \text{ N/m}^3$$

$$\text{radius}(r) = 0.60 \text{ m}$$

$$\omega = \frac{2\pi \times 400}{60}$$

$$= 41.90 \text{ rad/sec}$$

Now;

$$z = \frac{\omega^2 r^2}{2g}$$

$$= \frac{41.90^2 \times 0.60^2}{2 \times 9.81}$$

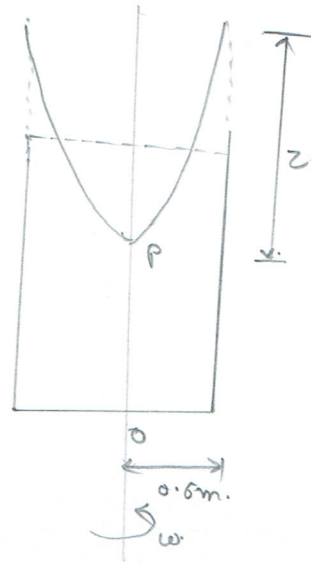
$$= 32.20 \text{ m}$$

$$\therefore \text{Pressure} = \gamma h$$

$$= 8.3 \times 10^3 \times 32.20$$

$$= 267428.90 \text{ Pa}$$

$$= 267.428 \text{ kPa}$$



311bhan

25. An open cylindrical vessel partially filled with water is 2ft in diameter and rotated about the axis, which is vertical. How many rpm would cause the water surface at the periphery to be 5ft higher than the water surface at the axis? what would be the necessary speed for the same conditions if the fluid were mercury?

Solution;

Given;

Diameter of cylinder (d) = 2ft.

Let N be the rotation per minute due to which water surface at end will be 5ft higher than centre of fall.

Now;

$$z_{\max} = \frac{\omega^2 r^2}{2g}$$

$$\text{or } 5 = \left(\frac{2\pi N}{60} \right)^2 \times \frac{1^2}{2 \times 32.18}$$

$$\text{or } \left(\frac{2\pi N}{60} \right)^2 = 321.8$$

$$\text{or } \frac{2\pi N}{60} = 17.93$$

$$\therefore N = 171.25 \text{ rpm}$$

There is no change in speed when mercury is used since it does not depend upon the nature of the liquid.

3116hax

14. A U tube system as shown in the figure below is rotated about axis AA at 10 rpm. Determine the levels in column C₁, C₂ and C₃ in the new position of equilibrium. Determine also the levels in the tubes, if the axis of rotation is BB instead of AA. Assume the tubes to be long enough not to allow spillage.

Soln:

Here, for the rotation about AA:

Let z_1, z_2 and z_3 be the heights of liquid in columns C₁, C₂ and C₃ respectively then;

$$z_1 = \frac{\omega^2 r^2}{2g} = \frac{\left(\frac{2\pi \times 10}{60}\right)^2 \times 0.5^2}{2 \times 9.81} = \frac{1.047^2 \times 0.5^2}{2 \times 9.81} = 0.01397 \text{ m.}$$

Same height will be raised in the column C₃.

Height raised in column C₂ will be zero.

Now; $z_1 + z_2 + z_3 = 2 \times 0.01397 = 0.0279 \text{ m.}$

So, level of liquid in column C₁ = $0.01397 + \left(0.3 - \frac{0.0279}{3}\right)$

$$= 0.3047 \text{ m.}$$

$$= 30.47 \text{ cm.}$$

Same level will be in column C₃.

level of liquid in column C₂ = $0.3 - \frac{0.0279}{3}$

$$= 0.2906 \text{ m.}$$

$$= 29.06 \text{ cm.}$$

For the rotation about BB;

let the same assumption z_1, z_2, z_3 as in above;

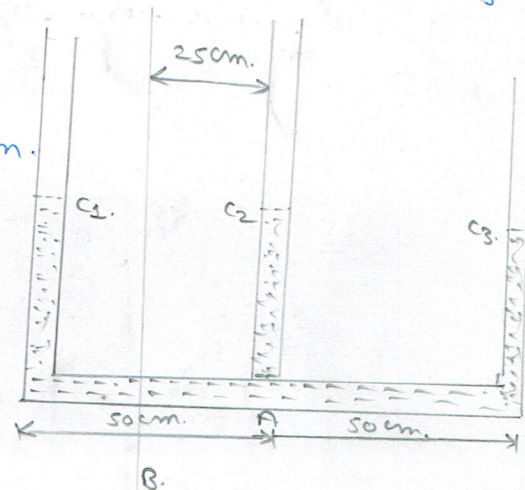
$$z_1 = \frac{1.047^2 \times 0.25^2}{2 \times 9.81} = 3.5 \times 10^{-3} \text{ m}$$

same height will be raised in column C₂ i.e. $z_1 = z_2$.

$$z_3 = \frac{1.047^2 \times 0.75^2}{2 \times 9.81} = 0.0314 \text{ m.}$$

Now; $z_1 + z_2 + z_3 = (3.5 \times 10^{-3}) \times 2 + 0.0314 = 0.0384 \text{ m.}$

So, level of liquid in column C₁ = level of liquid in column C₂ = $3.5 \times 10^{-3} + \left(0.3 - \frac{0.0384}{3}\right)$



311Bha

$$= 0.2907 \text{ m} = 29.07 \text{ cm.}$$

$$\text{level of liquid in column } C_3 = 0.0314 + \left(0.3 - \frac{0.0384}{3} \right).$$

$$= 0.3186 \text{ m.}$$

$$= 31.86 \text{ cm.}$$

16. The U-tube of above solved example (3.71) is rotated about the centre of the horizontal leg so that the pressure at the centre is zero. Calculate ω if $L=60 \text{ cm}$ and 40 cm .

Solution;

$$\text{For } L = 60 \text{ cm} = 0.6 \text{ m.}$$

we obtain;

$$\text{radius } (R) = 0.45 \text{ m.}$$

for the pressure at the centre to be zero, we have,

$$Z_{\max} = \frac{\omega^2 R^2}{2g}.$$

$$\text{or, } 0.6 = \frac{\omega^2 \times 0.45^2}{2 \times 9.81}.$$

$$\text{or, } \omega^2 = 58.0644.$$

$$\therefore \omega = 7.62 \text{ rad/sec.}$$

$$\text{For } L = 40 \text{ cm} = 0.4 \text{ m.}$$

$$\text{we obtain radius } (R) = 0.3 \text{ m.}$$

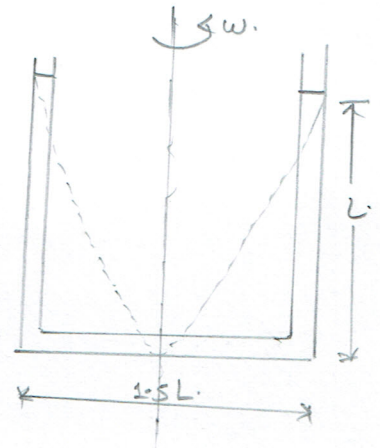
for the pressure at centre to be zero,

$$Z_{\max} = \frac{\omega^2 R^2}{2g}.$$

$$\text{or, } 0.4 = \frac{\omega^2 \times 0.3^2}{2 \times 9.81}.$$

$$\text{or, } \omega^2 = 87.2356.$$

$$\therefore \omega = 9.34 \text{ rad/sec.}$$



311bha

17. For solved example (3.73) determine the pressure at point A for a rotation speed of 5 rad/sec .

Solution;

Let the cylinder be open and long enough to hold the water.

Given;

$$\omega = 5 \text{ rad/sec.}$$

$$z = \frac{\omega^2 r^2}{2g}$$

$$= \frac{5^2 \times 0.6^2}{2 \times 9.81}$$

$$= 0.458 \text{ m}$$

Volume = C.

$$\pi \times 0.6^2 \times 0.6 = \frac{1}{2} \times 0.458 \times \pi \times 0.6^2 + \pi \times 0.6^2 \times x$$

$$\Rightarrow x = 0.372 \text{ m}$$

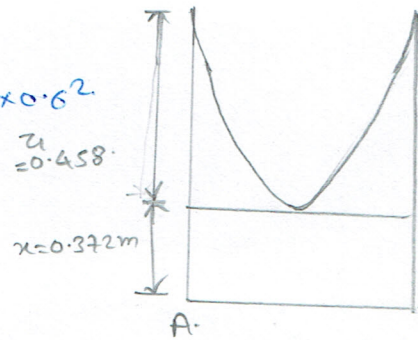
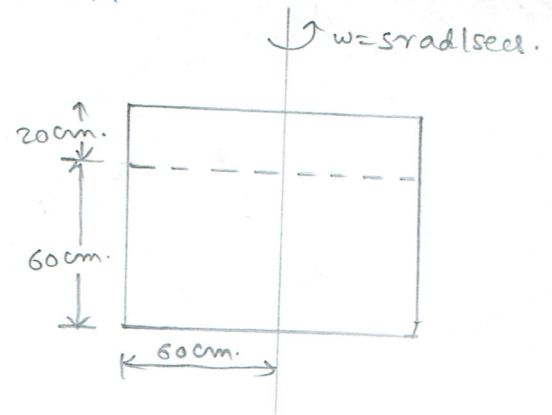
So,

Pressure at A is;

$$P_A = (0.458 + 0.372) \times 9.81 + \frac{1 \times 5^2 \times 0.6^2}{2}$$

$$= 8.14 + 4.5$$

$$= 12.64 \text{ kPa.}$$



Bilbhar

Tutorial 5

Hydrokinematics and Hydrodynamics

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Abhash

1. An idealized flow is given by $V = 2x^3\vec{i} - 3x^2y\vec{j}$. Is this flow steady or unsteady? Is it two or three dimensional? make calculation for the velocity, local acceleration, and convective acceleration of a fluid particle in this flow at point $P(x, y, z) = (2, 1, 3)$.

Solution:

Given: $V = 2x^3\vec{i} - 3x^2y\vec{j}$.

Since, V is not the function of time, the flow is steady.

The flow is two dimensional since it is independent of z -direction.

Here, $u = 2x^3$ $v = -3x^2y$.

At $(2, 1, 3)$ $u = 2 \times 8 = 16$ $v = -12$

i.e. $\vec{V} = 16\vec{i} - 12\vec{j}$

So, $V = \sqrt{16^2 + 12^2}$
 $= 20 \text{ units.}$

Now;

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = 2x^3 \cdot 6x^2 - 3x^2y \times 0$$
$$= 12x^5.$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = 2x^3(-3xy) - 3x^2y(-3x^2)$$
$$= -12x^4y + 9x^4y$$
$$= -3x^4y$$

$$\therefore a_x = 12x^5$$

$$a_y = -3x^4y$$

311bhar

2. The velocity component in a flow field are known to be

$$u = 6xz + y^2z + 15; \quad v = 3xy^2 + t^2 + y; \quad w = z + 3ty.$$

Classify the velocity field as steady or unsteady, uniform or non-uniform and one, two or three dimensional. What is the velocity vector at a point P (3, 2, 4) at time $t = 3$ sec. Also calculate the magnitude of this vector at this point and time.

Solution;

$$\frac{du}{dx} = 6z \quad \frac{dv}{dy} = 6xy + 1 \quad \frac{dw}{dz} = 0.$$

Now;

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 6z + 6xy + 1 \neq 0.$$

$\therefore$ The flow is unsteady and non-uniform, three-dimensional flow.

For P (3, 2, 4) and $t = 3$ sec.

$$\begin{aligned} u &= 6 \times 3 \times 3 + 2^2 \times 4 + 15 \\ &= 54 + 16 + 15 \\ &= 85. \end{aligned}$$

$$\begin{aligned} v &= 3 \times 3 \times 2^2 + 3^2 + 2 \\ &= 36 + 9 + 2 \\ &= 47. \end{aligned}$$

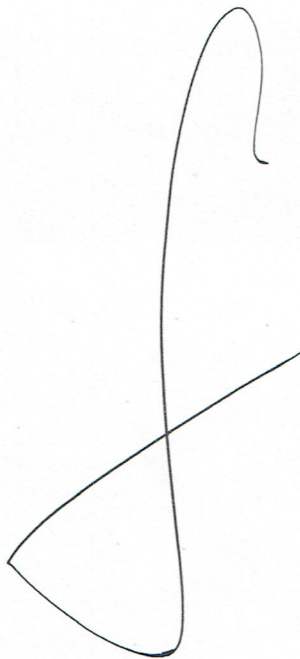
$$\begin{aligned} w &= z + 3 \times 3 \times 2 \\ &= 20. \end{aligned}$$

$$\therefore \vec{V} = 85\vec{i} + 47\vec{j} + 20\vec{k}.$$

$$|\vec{V}| = \sqrt{85^2 + 47^2 + 20^2}.$$

$$\text{or, } V = \sqrt{9834}.$$

$$\therefore V = 99.2 \text{ m/s.}$$



Zilbhan

3. For a three dimensional flow field described by;

$$V = (y^2 + z^2)\vec{i} + (x^2 + z^2)\vec{j} + (x^2 + y^2)\vec{k}.$$

Find at (1, 2, 3) the components of acceleration and the component of rotation.

Solution;

Here;

$$u = y^2 + z^2 \quad v = x^2 + z^2 \quad w = x^2 + y^2.$$

Now;

$$\begin{aligned} a_x &= u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \\ &= (y^2 + z^2) \cdot 0 + (x^2 + z^2) \cdot 2y + (x^2 + y^2) \cdot 2z \\ &= (1 + 9) \times 4 + (5 \times 2 \times 3) \end{aligned}$$

$$\therefore a_x = 70 \text{ units.}$$

$$\begin{aligned} a_y &= u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \\ &= (y^2 + z^2) \cdot 2x + (x^2 + z^2) \times 0 + (x^2 + y^2) \times 2z \\ &= 13 \times 2 + 30 \end{aligned}$$

$$\therefore a_y = 56 \text{ units.}$$

And;

$$\begin{aligned} a_z &= u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \\ &= (y^2 + z^2) \cdot 2x + (x^2 + z^2) \cdot 2y \\ &= 26 + 40 \end{aligned}$$

$$\therefore a_z = 66 \text{ units.}$$

For rotational components;

$$\omega_x = \frac{1}{2} \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) = \frac{1}{2} (2y - 2z) = 2 - 3 = -1 \text{ units.}$$

$$\omega_y = \frac{1}{2} \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) = \frac{1}{2} (2z - 2x) = 3 - 1 = 2 \text{ units.}$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{1}{2} (2x - 2y) = x - y = -1 \text{ units.}$$

Alibhah

4. Do the following velocity components represent physically possible flows?

• $u = x^2 + z^2 + 5$; $v = y^2 + z^2$; $w = 4xy$.

solution;

$$\frac{du}{dx} = 2x, \quad 2x \frac{dv}{dy} = 2y, \quad \frac{dw}{dz} = 0.$$

$$\text{or, } \frac{dv}{dy} = \frac{y}{\sqrt{y^2 + z^2}}.$$

$$\therefore \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 2x + \frac{y}{\sqrt{y^2 + z^2}} + 0 \neq 0.$$

So, the given velocity components, doesn't represent physically possible flow.

• $u_r = \left[1 - \frac{A}{r^2}\right] \cos \theta$, $u_\theta = \left[1 + \frac{A}{r^2}\right] \sin \theta$.

solution;

$$\frac{du}{dr} = \cos \theta \left[\frac{2A}{r^3} \right], \quad \frac{du}{d\theta} = \left[1 + \frac{A}{r^2} \right] \cdot \cos \theta.$$

$$\therefore \frac{du}{dr} + \frac{du}{d\theta} = \frac{2A \cos \theta}{r^3} + \cos \theta \left[1 + \frac{A}{r^2} \right] \neq 0.$$

So, the flow is not possible.

• $u_r = \frac{m}{4\pi r}$; $u_\theta = 0$; $u_z = 0$.

solution;

$$\frac{du}{dr} = \frac{m}{4\pi} \cdot \frac{(-1)}{r^2}.$$

So, the flow is not possible.

• $u = 5x\vec{i} + (3y + ty^2)\vec{j}$.

solution;

$$u = 5x, \quad v = 3y + ty^2.$$

$$\frac{du}{dx} = 5, \quad \frac{dv}{dy} = 3 + 2ty.$$

$$\therefore \frac{du}{dx} \neq \frac{dv}{dy} \neq 0.$$

So, the flow is not possible.

Alibhara

Since, $H_2 > H_1$, water flows from 20m pipe to 12m pipe.

4. A pipe 300m long has a slope of 1 in 100 and tapers from 1.25m diameter at the high end to 0.625m diameter at low end. Find the pressure at low end if the flow rate is 100 lps and pressure at high end is 1 kgf/cm^2 .

Solution;

Given;

Length of the pipe = 300m.

$$z_2 = 0 \text{m.}$$

$$z_1 = 3 \text{m.}$$

$$d_1 = 1.25 \text{m.}$$

$$d_2 = 0.625 \text{m.}$$

$$Q = 100 \text{ lps} = 100 \times 10^{-3} \text{ m}^3/\text{sec.}$$

$$P_1 = 1 \text{ kgf/cm}^2 = 98100 \text{ N/m}^2$$

Now;

$$v_1 = \frac{Q}{A_1} = \frac{100 \times 10^{-3}}{\frac{\pi \times (1.25)^2}{4}} = 0.08145 \text{ m/s.}$$

$$v_2 = \frac{Q}{A_2} = \frac{100 \times 10^{-3}}{\frac{\pi \times (0.625)^2}{4}} = 0.3258 \text{ m/s.}$$

Using Bernoulli's equation;

$$H_1 = H_2.$$

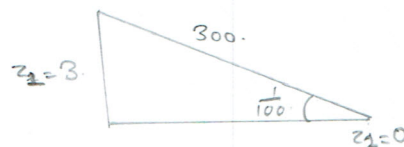
$$\text{or, } z_1 + \frac{v_1^2}{2g} + \frac{P_1}{\gamma} = z_2 + \frac{v_2^2}{2g} + \frac{P_2}{\gamma}$$

$$\text{or, } 3 + \frac{0.08145^2}{2 \times 9.81} + \frac{P_1}{9810} = 0 + \frac{0.3258^2}{2 \times 9.81} + \frac{P_2}{9810}$$

$$\text{or, } 3 + 3.38 \times 10^{-4} + 10 = 5.41 \times 10^{-3} + \frac{P_2}{9810}$$

$$\text{or, } 1.299 \times 10^4 = \frac{P_2}{9810}$$

$$\therefore P_2 = 1.2748 \times 10^5 \text{ N/m}^2.$$



Zillbhar

5. water flows through a horizontal 150 mm diameter pipe under the pressure of 4.14 bars. Assuming no losses, what is the flow if the pressure at 75 mm diameter reduction is 1.38 bar.

Solution;

Given;

$$d_1 = 150 \times 10^{-3} \text{ m.}$$

$$d_2 = 75 \times 10^{-3} \text{ m.}$$

$$P_1 = 4.14 \times 1.01 \times 10^5 \text{ Pa.}$$
$$= 4.1814 \times 10^5 \text{ Pa.}$$

$$P_2 = 1.39 \times 10^5 \text{ Pa.}$$

we have;

$$Q_1 = Q_2$$

$$A_1 V_1 = A_2 V_2$$

$$\text{or, } \frac{\pi \times (150 \times 10^{-3})^2}{4} \times V_1 = \frac{\pi \times (75 \times 10^{-3})^2}{4} \times V_2$$

$$\text{or, } 2.25 \times 10^{-2} V_1 = 5.625 \times 10^{-3} V_2$$
$$\therefore V_2 = 4 V_1$$

Now;

using Bernoulli's equation;

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\text{or, } \frac{4.1814 \times 10^5}{9810} + \frac{V_1^2}{2g} = \frac{1.39 \times 10^5}{9810} + \frac{16 V_1^2}{2g}$$

$$\text{or, } 28.4546 = \frac{16 V_1^2}{2g}$$

$$\therefore V_1 = 6.1 \text{ m/s.}$$

$$Q = A_1 V_1$$

$$\text{or, } Q = \frac{\pi \times (150 \times 10^{-3})^2}{4} \times 6.1$$

$$\therefore Q = 0.107 \text{ m}^3/\text{sec.}$$

Bilbhar

7. A reservoir supplies water to a horizontal 0.15m pipe 244m long. The pipe flow full and discharges into atmosphere at the rate of $0.063 \text{ m}^3/\text{s}$. What is the pressure midway in the pipe, assuming the only lost head is 1.86cm in each 30m of length.

Solution;

Given;

$$Q = 0.063 \text{ m}^3/\text{sec.}$$

The mid-way of the pipe = 122m.

$$\text{loss from midway to the exit} = \frac{1.86 \times 10^{-2}}{30} \times 122 = 0.07565 \text{ m.}$$

Now,

$$\text{velocity} = \frac{Q}{A} = \frac{0.063}{\frac{\pi \times (0.15)^2}{4}} = 3.56 \text{ m/s.}$$

Given that;

$$H_{\text{midway}} - H_{\text{exit}} = 0.07565$$

$$\text{or, } \frac{P_1}{9810} + \frac{3.56^2}{2 \times 9.81} - 0 - \frac{3.56^2}{2 \times 9.81} = 0.0756.$$

$$\therefore P_1 = 0.0756 \times 9810 \\ = 741.63 \text{ Pa.}$$

8. A pipeline carrying oil of specific gravity of 0.87 changes in diameter from 200mm at a position A to 500mm diameter at position B which is 4m at a higher level. If the pressure at A and B are 1 kgf/cm^2 and 0.6 kgf/cm^2 respectively and discharge is 200 lps determine the loss of head and direction of flow.

Solution;

Given;

$$S_{\text{oil}} = 0.87.$$

$$d_A = 200 \times 10^{-3} \text{ m}$$

$$d_B = 500 \times 10^{-3} \text{ m.}$$

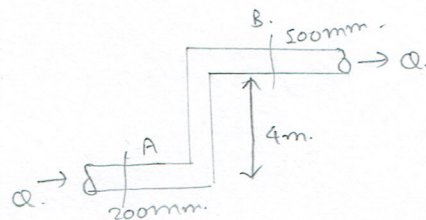
$$z_1 = 0.$$

$$z_2 = 4 \text{ m.}$$

$$P_A = 98100 \text{ N/m}^2.$$

$$P_B = 58860 \text{ N/m}^2.$$

$$Q = 200 \times 10^{-3} \text{ m}^3/\text{sec.}$$



$$\text{we have; } V_A = \frac{Q}{A_A} = \frac{200 \times 10^{-3} \times 4}{\pi \times (200 \times 10^{-3})^2} = 6.36 \text{ m/s.}$$

$$V_B = \frac{Q}{A_B} = \frac{200 \times 10^{-3} \times 4}{\pi \times (500 \times 10^{-3})^2} = 1.018 \text{ m/s.}$$

$$H_A = \frac{V_A^2}{2g} + \frac{P_A}{\gamma} = \frac{6.36^2}{2 \times 9.81} + \frac{98100}{0.87 \times 9810} = 12.56 \text{ m.}$$

$$H_B = z_B + \frac{V_B^2}{2g} + \frac{P_B}{\gamma} = 4 + \frac{1.018^2}{2 \times 9.81} + \frac{58860}{0.87 \times 9810} = 10.94 \text{ m.}$$

311bha

$$\begin{aligned}\text{Head loss} &= H_A - H_B \\ &= 13.56 - 10.94 \\ &= 2.62 \text{ m.}\end{aligned}$$

$\therefore H_A > H_B$, the oil flows from A to B.

9. A 75% efficient pump delivers $0.1 \text{ m}^3/\text{s}$ of water from a reservoir to a device at an elevation of 50 m above the reservoir. The pressure at the 8 cm diameter entrance to the device is 180 kPa . If the pipe loss coefficient is 5.6 times the velocity head, find the necessary power input to the pump.

Solution;

Given;

$$Q = 0.1 \text{ m}^3/\text{sec.}$$

$$d_1 = 0.08 \text{ m.}$$

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{\frac{\pi \times d_1^2}{4}} = \frac{0.1}{\frac{\pi \times 0.08^2}{4}} = 19.89 \text{ m/sec.}$$

$$z_1 = 50 \text{ m.}$$

Energy at the entrance;

$$H = z_1 + \frac{V_1^2}{2g} + \frac{P_1}{\gamma} + \text{Head loss}$$

$$= 50 + \frac{19.89^2}{2 \times 9.81} + \frac{180 \times 10^3}{9810} + 5.6 \times \frac{19.89^2}{2 \times 9.81}$$

$$= 201.42 \text{ m.}$$

$$\text{Power} = \gamma Q H.$$

$$= 9.81 \times 0.1 \times 201.5 \times 1000$$

$$= 197671.5 \text{ W}$$

Since the efficiency of the pump = 75%

$$\frac{197671.5 \text{ W}}{0.75}$$

$$\therefore \text{Power (Net)} = \frac{75}{100} \times 197671.5 \text{ W}$$

$$= \frac{1}{2} \times 197671.5 \text{ W}$$

$$= 263562 \text{ W}$$

$$= 263.562 \text{ kW.}$$

Bilbhar

10. A close tank (elevation 75m) at a pressure of 5 bar supplies water to a reservoir (elevation 45m) through on the way to a pipeline. A turbine (elevation 30m) is located which extracts a head of 60m. The pipeline from the tank to turbine is 0.30m in diameter and that from the turbine to reservoir is 0.60m in diameter. If the losses of head in the 1st and 2nd section are 2 and 3 times the corresponding velocity heads, respectively, determine
 a) the flow rate and b) pressure head at the exit of turbine.

Solution;

Elevation of tank (z_1) = 75m.

Pressure at tank (P_1) = 5 bar = 500 kPa.

Here,

$$A_1 = \pi \times (0.15)^2 = 70.68 \times 10^{-2} \text{ m}^2$$

$$A_2 = \pi \times 0.3^2 = 2.83 \times 10^{-1} \text{ m}^2$$

Using Bernoulli's Equation;

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + h_{\text{loss}(1-2)} + H_T + h_{\text{loss}(T-2)}$$

$$75 + \frac{500}{9.81} + \frac{Q^2}{2 \times 9.81 \times (70.68 \times 10^{-2})^2} = 45 + 0 + \frac{Q^2}{2 \times 9.81 \times (2.83 \times 10^{-1})^2} + Q^2 \left[\frac{2}{2g \times (70.68 \times 10^{-2})^2} + \frac{3}{2g \times 0.283^2} \right] + 60$$

$$20.97 = Q^2 \left[\frac{1}{1.568} + \frac{3}{1.568} + \frac{2}{0.098} - \frac{1}{0.098} \right]$$

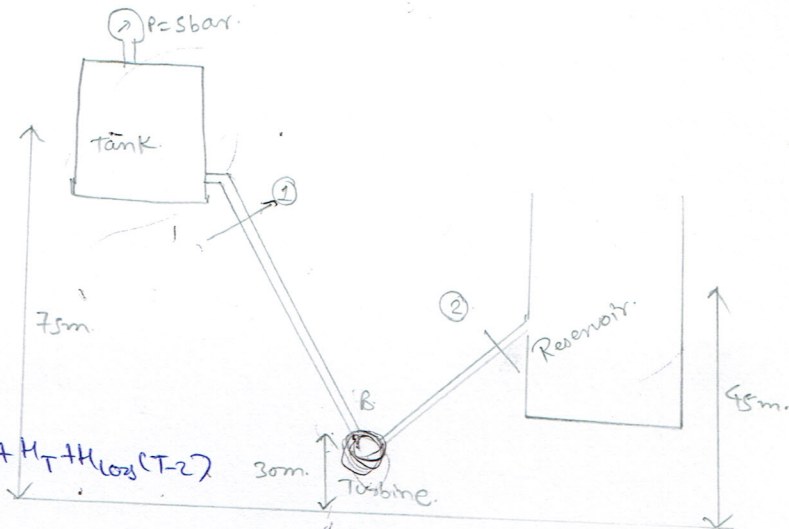
$$\Rightarrow Q = 1.282 \text{ m}^3/\text{sec}$$

Also;

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1^2}{2g} = z_3 + \frac{P_3}{\gamma} + h_{f-3} + h_T$$

$$75 + \frac{500}{9.81} + 16.77 = 30 + P + 1.048 + 2 \times 16.77 + 60$$

$$\therefore P = 18.15 \text{ m}$$



311bha

11. calculate the flow of water through a 40cm by 15cm venturimeter when the differential gauge connected to the inlet end of the meter and its throat shows 25cm of mercury. Take $C_d = 0.9$.

solution;

Given;

$$\text{Area } (A_1) = 40\text{cm} \times 15\text{cm} \\ = 600\text{cm}^2 \\ = 0.6\text{m}^2.$$

$$\text{Area } (A_1) = \pi \times \frac{(0.40)^2}{4} = 0.1257\text{m}^2.$$

$$C_d = 0.9.$$

$$\text{Area } (A_2) = \pi \times \frac{(0.15)^2}{4} = 0.0176\text{m}^2.$$

$$\sqrt{2gh} = \sqrt{2 \times 9.8 \times \frac{25}{100}} = \sqrt{4.905} = 2.21.$$

Here;

$$Q_{th} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{0.1257 \times 0.0176 \times 2.21}{\sqrt{0.1257^2 - 0.0176^2}} = 0.03928.$$

$$Q_{act} = C_d \times Q_{th} = 0.9 \times 0.03928 \\ = 0.03535\text{m}^3/\text{sec}.$$

12. A venturimeter with a throat diameter of 100mm is fitted in a vertical pipeline of 200mm diameter with oil of sp-gravity 0.88 flowing upwards at a rate of 0.06m/sec. The pressure gauges fitted at tapping points, one at the throat and the other in the inlet pipe 320mm below the throat. The difference between the two gauge pressure readings is 28kN/m². Taking $C_d = 0.96$, determine i) the discharge of oil, and ii) the difference in level in two limbs of mercury manometer.

solution;

Applying Bernoulli's equation between inlet and throat.

$$z_i + \frac{v_i^2}{2g} + \frac{P_i}{\gamma} = z_t + \frac{v_t^2}{2g} + z_t.$$

$$\text{or, } \frac{P_i - P_t}{\gamma} + (z_i - z_t) = \frac{v_t^2}{2g} - \frac{v_i^2}{2g}.$$

$$\text{or, } \frac{28000}{880 \times 9.81} - 0.32 = \frac{v_t^2}{2g} - \frac{v_i^2}{2g}.$$

$$\therefore \frac{v_t^2}{2g} - \frac{v_i^2}{2g} = 2.92 - \text{i)}$$

Bilbhan

Now,

using equation of continuity;

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1 v_1}{A_2} = \frac{0.0314 v_1}{0.00789 v_1} = 4 v_1 \quad \text{--- (i)}$$

Solving i) and ii),

$$(4 v_1)^2 - v_1^2 = 2.92 \times 2 \times 9.81$$

$$15 v_1^2 = 57.29$$

$$\therefore v_1 = 1.95 \text{ m/sec}$$

$$\text{So discharge } Q_{th} = A_1 v_1 = 0.0314 \times 1.95 = 0.0612 \text{ m}^3/\text{sec}$$

$$\begin{aligned} \text{Actual discharge } (Q_{act}) &= 0.96 \times 0.0612 \\ &= 0.0588 \text{ m}^3/\text{sec} \\ &= 58.8 \text{ lps} \end{aligned}$$

Now,

$$h = \frac{P_1 - P_2}{\gamma} + (z_1 - z_2) = \frac{28000}{880 \times 9.81} = 0.32 = 2.92$$

And let, 'x' be the difference in level in two limbs of mercury then,

$$2.92 = x \left[\frac{13.6}{0.88} - 1 \right]$$

$$\therefore x = 0.20 \text{ m}$$

Bilbhat

13. A (5x2.5) cm venturimeter with $C_d = 0.98$ is to be replaced by an orifice meter with $C_d = 0.6$. If both the meters are given the same differential mercury manometer reading for a discharge of 11 lps and the inlet diameter is to remain 5cm, what should be the diameter of orifice. Take $C_c = 1$.

Solution;

Given;

$$d_1 = 5 \times 10^{-2} \text{ m.}$$

$$d_2 = 2.5 \times 10^{-2} \text{ m.}$$

$$C_d = 0.98$$

$$Q_{\text{act}} = 11 \times 10^{-3} \text{ m}^3/\text{sec.}$$

Now;

$$Q_{\text{act}} = \frac{C_d \times A_1 \times A_2 \times \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$$

$$11 \times 10^{-3} = \frac{0.98 \times \pi \times \frac{(5 \times 10^{-2})^2}{4} \times \pi \times \frac{(2.5 \times 10^{-2})^2}{4} \times \sqrt{2 \times 9.81 \times h}}{\sqrt{\left(\pi \times \frac{(5 \times 10^{-2})^2}{4}\right)^2 - \left(\pi \times \frac{(2.5 \times 10^{-2})^2}{4}\right)^2}}$$

$$11 \times 10^{-3} = \frac{9.45 \times 10^{-7} \times \sqrt{2 \times 9.81 \times h}}{\sqrt{3.85 \times 10^{-6} - 2.41 \times 10^{-7}}}$$

$$11 \times 10^{-3} = \frac{9.45 \times 10^{-7} \sqrt{2 \times 9.81 \times h}}{1.899 \times 10^{-3}}$$

$$\text{or, } 2.211 \times 10^1 = \sqrt{2 \times 9.81 \times h}$$

$$\therefore h = 15.4 \text{ m.}$$

Now, for orifice;

$$Q_{\text{act}} = \frac{C_d \times a_1 a_2 \times \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$11 \times 10^{-3} = \frac{0.6 \times \frac{\pi}{4} \times 0.05^2 \times \frac{\pi}{4} \times d^2 \times \sqrt{2gh}}{\sqrt{\left(\frac{\pi}{4} \times 0.05^2\right)^2 - \left(\frac{\pi}{4} d^2\right)^2}}$$

$$11 \times 10^{-3} = \frac{1.5 \times 10^{-3} d^2 \times 17.388}{\sqrt{6.25 \times 10^{-6} \times d^4}}$$

$$\therefore d = 0.03116 \text{ m.}$$

3116ha4

15. A Pitot static probe is used to measure the flow of water in a 5cm diameter pipe. If the mean velocity is 5m/s and Pitot Static Probe is connected across mercury fluid differential manometer, what should be the difference of mercury columns?

solution;

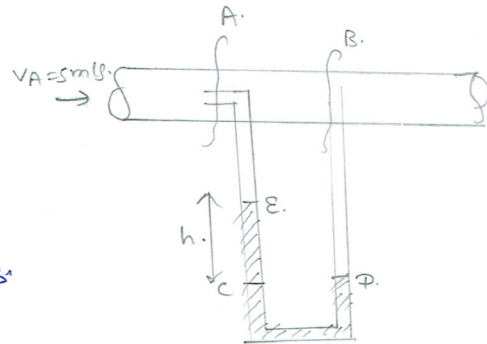
given;

diameter of the pipe = 5cm.

now, $H_A = H_B$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B$$

$$\text{or, } \frac{P_B}{\gamma} - \frac{P_A}{\gamma} = \frac{V_A^2}{2g} \quad \text{--- (i)}$$



let 'h' be the difference in height of mercury column, then;

$$P_C = P_D$$

$$\frac{P_A}{\gamma} + 13.6h = \frac{P_B}{\gamma} + h$$

$$\text{or, } 12.6h = \frac{P_B}{\gamma} - \frac{P_A}{\gamma}$$

using (i).

$$12.6h = \frac{V_A^2}{2g}$$

$$\text{or, } h = \frac{5^2}{12.6 \times 2 \times 9.81}$$

$$\therefore h = 0.101 \text{ m.}$$

16. A 30cm x 15cm venturimeter is provided in a vertical pipeline carrying oil of sp.gr. 0.9 and the flow being upwards. The differential U-tube mercury manometer shows a gauge reading of 25cm. Calculate i) the discharge of the oil; ii) the pressure difference between entrance and throat section taking $C_d = 0.98$.

solution;

Here, $Z_1 - Z_2 = 0.3 \text{ m.}$

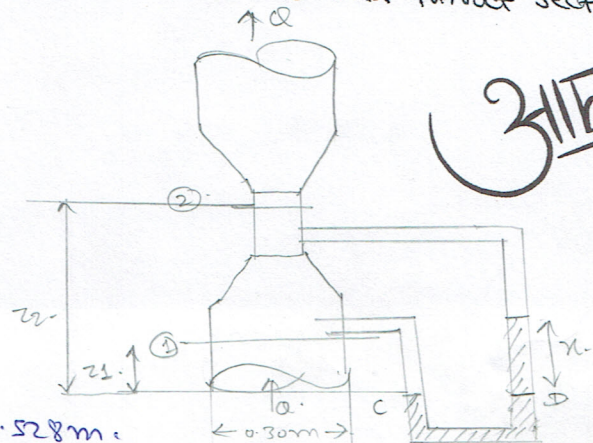
$$A_1 = \pi \times 0.15^2 = 0.07068 \text{ m}^2$$

$$A_2 = \pi \times 0.075^2 = 0.01767 \text{ m}^2$$

$$C_d = 0.98 \text{ and } x = 0.25 \text{ m.}$$

So,

$$h = x \left[\frac{\gamma_{Hg}}{\gamma} - 1 \right] = 0.25 \left[\frac{13.6}{0.9} - 1 \right] = 3.528 \text{ m.}$$



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we have;

$$Q = \frac{C_d A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} = \frac{0.98 \times 0.07068 \times 0.01767 \times \sqrt{2 \times 9.81 \times 3.528}}{\sqrt{(0.07068)^2 - (0.01767)^2}}$$

$$= 0.1487 \text{ m}^3/\text{sec.}$$

Also; pressure at C = pressure at D.

$$P_1 + \gamma z_1 = P_2 + (z_2 - x) \gamma + x \gamma_{Hg}$$

$$\Rightarrow P_1 - P_2 = \gamma (z_2 - x - z_1) + \gamma_{Hg} x$$

$$= 0.9 \times 9810 \times (0.3 - 0.25) + 13.6 \times 9810 \times 0.25$$

$$= 33795.45 \text{ Pa.}$$

$$= 33.79545 \text{ kPa.}$$

17. A venturimeter is introduced in a 300mm diameter horizontal pipeline carrying water under a pressure of 150 kN/m². The throat diameter of the meter is 100mm and the pressure at the throat is 400mm of mercury below atmosphere. If 3% of the differential pressure is lost between the inlet and outlet throat, determine the flow rate in the pipe.

Solution;

$$A_1 = \pi \times \frac{(0.3)^2}{4} = 0.0706 \text{ m}^2$$

$$A_2 = \pi \times \frac{(0.1)^2}{4} = 7.854 \times 10^{-3} \text{ m}^2$$

$$P_2 = -400 \text{ mm gauge.}$$

$$= -53.36 \text{ kN/m}^2$$

$$P_1 = 150 \text{ kN/m}^2$$

$$\therefore P_1 - P_2 = 150 - (-53.36) = 203.36 \text{ kPa.}$$

Head loss from section 1 to 2 is

$$H_{\text{loss}} = 3\% \text{ of } \frac{P_1 - P_2}{\gamma} = \frac{203.36}{9.81} \times \frac{3}{100} = 0.622 \text{ m.}$$

Using Bernoulli's equation;

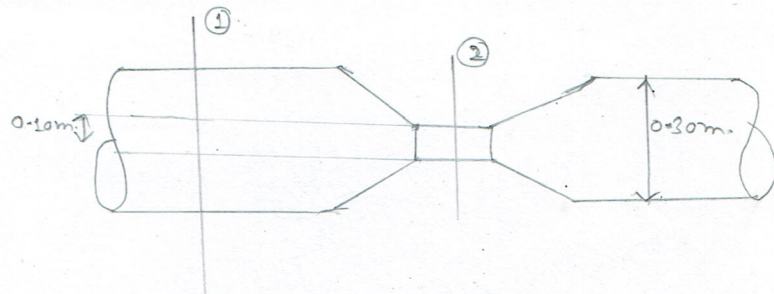
$$H_1 = H_2 + H_{\text{loss.}}$$

$$\therefore \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + 0.622$$

$$\therefore \frac{150}{9.81} + \frac{Q^2}{A_1^2 \times 2g} = \frac{-53.36}{9.81} + \frac{Q^2}{A_2^2 \times 2g} + 0.622$$

$$\therefore Q^2 \left[\frac{1}{A_2^2} - \frac{1}{A_1^2} \right] = 20.108 \times 2g$$

$$\therefore Q = 0.157 \text{ m}^3/\text{sec.}$$



Bilbhan

18. A 15cm diameter vertical pipe is connected to a 10cm diameter vertical pipe with reducing socket. The pipe carries water with the rate of 100 lps. At point 1 in the 15cm pipe, the gauge pressure is 250 kPa. At point 2 in the 10cm pipe, located 10m below point 1, the gauge pressure is 175 kPa. Find a) whether the flow is upwards or downwards. b) the head loss between the two points.

Solution;

Here;

$$P_1 = 250 \text{ kPa.}$$

$$P_2 = 175 \text{ kPa.}$$

$$Q = 100 \text{ lps.}$$

$$= 0.1 \text{ m}^3/\text{sec.}$$

$$A_1 = \pi \times 0.075^2 = 0.0176 \text{ m}^2.$$

$$A_2 = \pi \times 0.05^2 = 7.85 \times 10^{-3} \text{ m}^2.$$

$$V_1 = \frac{Q}{A_1} = \frac{0.1}{0.0176} = 5.68 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.1}{7.85 \times 10^{-3}} = 12.73 \text{ m/s.}$$

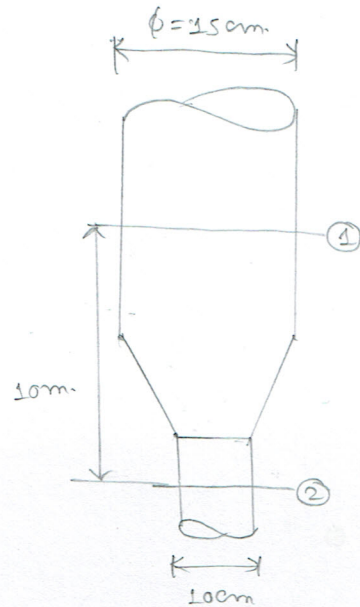
Using Bernoulli's equation;

$$H_1 = H_2 + H_{\text{loss}}$$

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + H_{\text{loss}}$$

$$\begin{aligned} \Rightarrow H_{\text{loss}} &= (z_1 - z_2) + \frac{1}{\gamma} (P_1 - P_2) + \frac{1}{2g} (V_1^2 - V_2^2) \\ &= 10 + \frac{1}{9.81} \times 75 + \frac{1}{2 \times 9.81} (5.68^2 - 12.73^2) \\ &= 11.03 \text{ m.} \end{aligned}$$

Since, loss is positive $\therefore H_1 > H_2$, so water flows downward.



19. A sharp edged orifice, 5cm in diameter, in the vertical side of a large tank discharges under head of 5m. If $C_c = 0.62$ and $C_v = 0.98$, determine a) the diameter at jet at vena-contracta, b) the velocity of the jet at the vena-contracta and c) the discharge.

Solution;

Given; diameter (d) = 0.05m.

$$H = 5 \text{ m.}$$

$$C_c = 0.62.$$

$$C_v = 0.98.$$

Let d_v be the diameter of the vena-contracta.

$$d_v^2 = C_c \times d^2 = 0.62 \times (5 \times 10^{-2})^2 = 0.00155.$$

$$\therefore d_v = 0.0394 \text{ m.}$$

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velocity of the jet at the vena-contracta.

$$v = C_v \times \sqrt{2gH}$$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 5}$$

$$= 9.71 \text{ m/s}$$

31bha

Discharge (Q) = AV = $\pi \times (2.5 \times 10^{-2})^2 \times 9.71$

$$= 0.012 \text{ m}^3/\text{sec}$$

20. A sharp-edged orifice has a diameter of 2.5 cm and C_v and C_c are 0.98 and 0.62, respectively. The jet drops 1 m in a horizontal distance of 2.65 cm. Determine the flow in m^3/sec and the head of the orifice.

Solution;

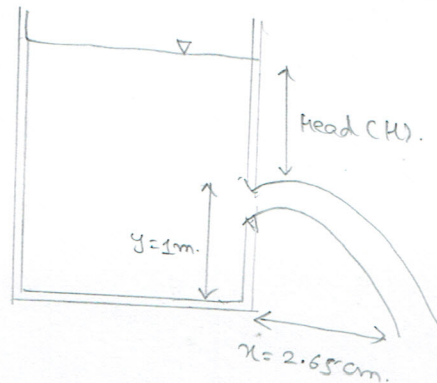
given, $x = 2.65 \text{ cm}$

$$= 2.65 \times 10^{-2} \text{ m}$$

$$\text{or } x = C_v \sqrt{4yH}$$

$$\text{or } 2.65 \times 10^{-2} = 0.98^2 \times 4 \times 1 \times H$$

$$\therefore H = 1.83 \times 10^{-4} \text{ m}$$



velocity at vena-contracta $(v) = C_v \sqrt{2gH}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times (1.83 \times 10^{-4})}$$

$$= 5.87 \times 10^{-2} \text{ m/s}$$

Diameter of vena-contracta $(d') = 0.62 \times 0.025^2$

$$= 0.0196 \text{ m}$$

$\therefore$ Discharge (Q) = AV = $\pi \times (9.8 \times 10^{-3})^2 \times 5.87 \times 10^{-2}$

$$= 1.78 \times 10^{-5} \text{ m}^3/\text{sec}$$

21. A tank has two identical orifices in one of its vertical sides. The upper orifice is 3 m below the water surface and lower one is 5 m below the water surface. If the value of C_v for each orifice is 0.96, find the point of intersection of two jets.

Solution;

Let the point where two jets intersect be at x m.

Then; we have;

$$\frac{x}{\sqrt{4y_1 H_1}} = \frac{x}{\sqrt{4y_2 H_2}}$$

$$\Rightarrow y_1 H_1 = y_2 H_2$$

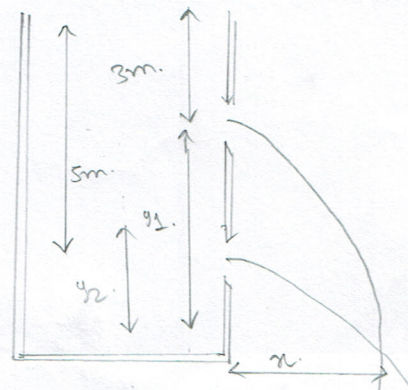
$$\text{or } (2 + y_2) H_1 = y_2 H_2$$

$$\text{or } (2 + y_2) \times 3 = y_2 \times 5$$

$$\therefore y_2 = 3 \text{ m}$$

Now, $x = C_v \sqrt{4y_2 H_2}$

$$= 0.96 \times \sqrt{60} = 7.40 \text{ m}$$



22. Find the discharge through a rectangular orifice 2.0m wide and 1.5m deep fitted to a water tank. The water level in the tank is 3.0m above the top edge of the orifice. Take $C_d = 0.62$; consider large orifice.

Solution;

If dA be an area of small strip and dQ be discharge through it then;

$$dQ = dA \times V \\ = b \times \sqrt{2gh} \times dh.$$

Integrating from H_1 to H_2 .

$$Q = \int_{H_1}^{H_2} b \sqrt{2g} h^{1/2} dh$$

$$Q_{th} = \int_3^{4.5} b \sqrt{2g} h^{1/2} dh.$$

$$= 2 \times \sqrt{2 \times 9.81} \times \frac{2}{3} [H_2^{3/2} - H_1^{3/2}]$$

$$= 2 \times \sqrt{2 \times 9.81} \times \frac{2}{3} [4.5^{1.5} - 3^{1.5}]$$

$$\therefore Q_{th} = 25.68 \text{ m}^3/\text{sec}.$$

So;

$$Q_{act} = C_d \times Q_{th} = 0.62 \times 25.68 = 15.92 \text{ m}^3/\text{sec}.$$

23. A rectangular orifice, 1.5m wide and 1.0m deep, is discharging water from a tank. If the water level in the tank is 3.0m above the top edge of the orifice, find the discharge through the orifice. Take the coeff. of discharge for the orifice as 0.6.

Solution;

Given; $b = 1.5 \text{ m}.$

$H_1 = 3 \text{ m}.$

$H_2 = 4 \text{ m}.$

Now;

$$Q_{th} = b \sqrt{2g} \times \frac{2}{3} [H_2^{3/2} - H_1^{3/2}]$$

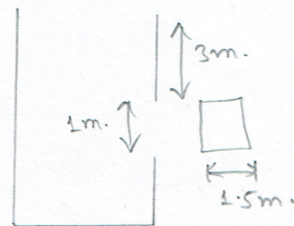
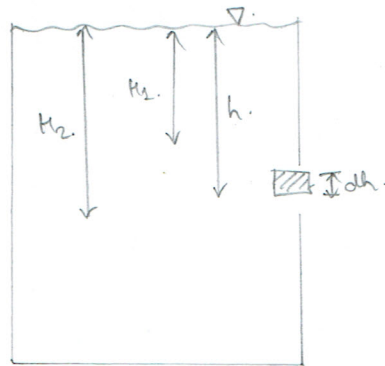
$$= 1.5 \times \sqrt{2 \times 9.81} \times \frac{2}{3} [4^{1.5} - 3^{1.5}]$$

$$= 12.42 \text{ m}^3/\text{sec}.$$

$$Q_{act} = Q_{th} \times C_d.$$

$$= 0.6 \times 12.42$$

$$= 7.45 \text{ m}^3/\text{sec}.$$



311bha

24. Water flows from a reservoir through a rectangular opening 2m high and 1.2m wide in the vertical face of a dam. Calculate the discharge when the free surface in the reservoir is 0.5m above the top of the opening assuming a coefficient of discharge of 0.64.

Solution;

Height of the reservoir (H_1) = 0.5m.

Height of bottom of reservoir (H_2) = 2.5m.

Then;

$$discharge (Q) = \frac{2}{3} \times \sqrt{2g} \times b \times [H_2^{3/2} - H_1^{3/2}].$$

$$[\because Q = \int_{H_1}^{H_2} b \sqrt{2g} h^{1/2} dh].$$

$$= 0.64 \times \frac{2}{3} \times \sqrt{2 \times 9.81} \times 1.2 [2.5^{1.5} - 0.5^{1.5}] \times 0.64.$$

$$= 8.16 \text{ m}^3/\text{sec}.$$

25. A vertical triangular orifice in the wall of a reservoir has base 0.9m long, 0.6m below its vertex and 1.2m below the water surface. Find the theoretical discharge.

Solution;

we have;

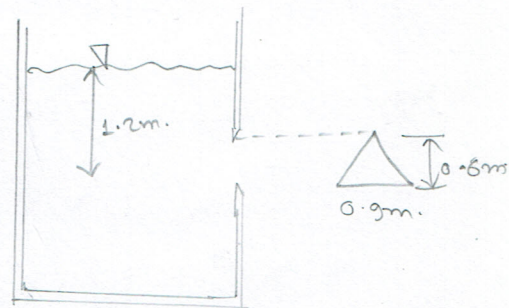
Discharge through a triangular orifice is;

$$Q = \frac{1}{3} \times b \times \sqrt{2g} \times (H_2^{3/2} - H_1^{3/2}).$$

$$= \frac{1}{3} \times 0.9 \times \sqrt{2 \times 9.81} \times (1.2^{1.5} - 0.6^{1.5})$$

$$= 1.329 \times [1.314 - 0.464].$$

$$= 1.12 \text{ m}^3/\text{sec}.$$



26. Water flows over a rectangular notch 1m wide at a depth of 15cm and afterwards passes through a triangular right angled notch. Taking $C_d = 0.6$ and $C_d = 0.59$ for rectangular and triangular notches respectively, find the head over the crest for the triangular notch.

Solution;

width of notch (b) = 1m.

Head over the crest (H) = 0.15m.

$C_{d1} = 0.6$.

$$\text{Then; } Q = C_{d1} \times \frac{2}{3} \times \sqrt{2g} \times H^{3/2} \times b$$

$$= 0.6 \times \frac{2}{3} \times \sqrt{2 \times 9.81} \times 0.15^{3/2} \times 1$$

$$= 0.103 \text{ m}^3/\text{sec}.$$

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For rt. angled Δ ; notch; $\frac{\theta}{2} = 45^\circ$.

we have to calculate the head over the ^{crest} notch; (h) ;
Here;

$$Q = C_{d2} \times \tan \frac{\theta}{2} \times \frac{8}{15} \sqrt{2g} H^{5/2}.$$

$$\text{or } 0.103 = 0.59 \times 1 \times \frac{8}{15} \times \sqrt{2 \times 9.81} \times h^{5/2}.$$

$$\text{or } h^{5/2} = 0.0738$$

$$\therefore h = 0.325 \text{ m.}$$

27. In an experiment on a 90° V-notch, the flow is collected in a 0.9 m diameter vertical cylindrical tank. It is found that the depth of water increases by 0.685 m in 16.8 seconds. when the head over the notch is 0.2 m . Determine the coeff. of discharge of the notch.

Solution;

$$\text{Discharge } (Q) = C_d \times \tan \frac{\theta}{2} \times \frac{8}{15} \sqrt{2g} \times H^{5/2}.$$

$$\text{for } 90^\circ, \text{ V-notch } \tan \frac{\theta}{2} = \tan 45^\circ = 1.$$

And;

$$Q = \frac{0.45^2 \times \pi \times 0.685}{16.8} = 0.026 \text{ m}^3/\text{sec}.$$

$$\text{or } 0.026 = C_d \times \frac{8}{15} \times \sqrt{2 \times 9.81} \times (0.2)^{2.5}.$$

$$\text{or } C_d = \frac{0.026}{0.04226}.$$

$$\therefore C_d = 0.615.$$

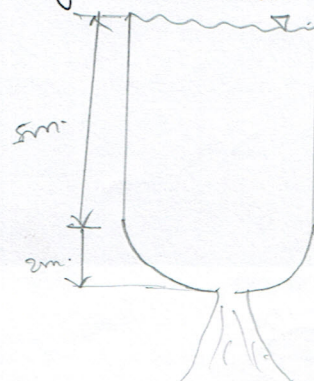
28. A cylindrical tank is having a hemispherical base. The height of cylindrical portion is 5 m and diameter is 4 m . At the bottom of this tank an orifice of diameter 20 cm is fitted. If the tank is initially full of water, find the time required to completely emptying the tank. Take $C_d = 0.6$.

Solution;

For Hemispherical part;

$$\begin{aligned} \text{Then } &= \frac{\pi}{C_d \times a \sqrt{2g}} \times \left[\frac{4}{3} \times R H_1^{1.5} - \frac{2}{5} H_1^{2.5} \right] \\ &= \frac{\pi}{0.6 \times \pi \times 0.12 \times \sqrt{2 \times 9.81}} \times \left[\frac{4}{3} \times 2 \times 2^{1.5} - \frac{2}{5} \times 2^{2.5} \right] \\ &= 198.66 \text{ sec.} \end{aligned}$$

And for the cylindrical part;



$$T_{cyl} = \frac{2A(\sqrt{H_2} - \sqrt{H_1})}{C_d \times a \times \sqrt{2g}}$$

$$= \frac{2 \times 7 \times 2^2 \times (\sqrt{7} - \sqrt{5})}{0.6 \times 7 \times 0.1^2 \times \sqrt{2 \times 9.81}}$$

$$= 370.7 \text{ sec.}$$

So, total time required to empty the tank is;

$$T = T_{hem} + T_{cyl} = 198.66 + 370.7 \\ = 569.4 \text{ sec.}$$

$$\therefore T = \frac{569.4}{60} = 9 \text{ min } 29 \text{ sec.}$$

29. A cylindrical tank of internal diameter 0.6m, length 1.5m and axis vertical has a 5cm diameter sharp edged orifice ($C_d = 0.6$) in the bottom, open to atmosphere. The tank is open at the top and empty. If liquid is admitted into the tank from above at a constant rate of 14 lps, how long will it take to just fill the tank? How much liquid will escape through the orifice during that period?

Solution;

$$\text{Area of tank } (A) = \frac{\pi}{4} \times 0.6^2 = 0.2827 \text{ m}^2$$

$$\text{Area of orifice } (a) = \frac{\pi}{4} \times 0.05^2 = 0.00196 \text{ m}^2$$

Time required to fill the tank is;

$$t = \frac{2A}{C_d^2 \cdot 2g \cdot a^2} \left[Q_i \cdot \ln \frac{Q_i - K\sqrt{H_2}}{Q_i - K\sqrt{H_1}} + K(\sqrt{H_2} - \sqrt{H_1}) \right]$$

$$\text{Here; } K = C_d \cdot a \sqrt{2g}$$

$$= 0.6 \times 0.00196 \times \sqrt{2 \times 9.81} = 0.0052$$

$$\therefore t = \frac{2A}{K^2} \left[Q_i \ln \frac{Q_i - K\sqrt{H_2}}{Q_i - K\sqrt{H_1}} + K(\sqrt{H_2} - \sqrt{H_1}) \right]$$

$$= \frac{2 \times 0.2827}{0.0052^2} \left[0.0014 \ln \left[\frac{0.0014 - 0.0052\sqrt{1.5}}{0.0014 - 0.0052\sqrt{0}} \right] + 0.0052(\sqrt{1.5} - \sqrt{0}) \right]$$

$$\therefore t = 44.46 \text{ sec}$$

$$\text{Here; Inflow volume} = Q_i \times t = 0.014 \times 44.46 = 0.6224$$

$$\text{Volume of water in the tank} = A \times H_2 = 0.2827 \times 1.5 = 0.424 \text{ m}^3$$

$$\text{Liquid escaped through orifice} = 0.6224 - 0.424 = 0.1984 \text{ m}^3$$

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Tutorial 6

Flow Measurement and Momentum Principle

by
Abhash Acharya

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1. A nozzle and hose are attached to the ladder of a fire truck. What force is needed to hold a nozzle supplied by a 9 cm. diameter hose with a pressure of 2000 kPa? The nozzle outlet diameter is 3 cm.

Solution;

Applying continuity equation;

$$A_1 V_1 = A_2 V_2 \Rightarrow \pi \frac{d_1^2}{4} V_1 = \pi \frac{d_2^2}{4} V_2$$

$$\therefore V_1 = \left(\frac{d_2}{d_1} \right)^2 \times V_2$$

Applying Bernoulli equation at (i) and (ii).

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\text{or, } \frac{2000 \times 10^3}{9810} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\text{or, } V_2^2 - V_1^2 = 4000$$

$$\text{or, } V_2^2 \left[1 - \left(\frac{d_2}{d_1} \right)^4 \right] = 4000$$

$$\Rightarrow V_2 = \sqrt{\frac{4000}{1 - \left(\frac{0.03}{0.09} \right)^4}}$$

$$= 63.64 \text{ m/s}$$

$$V_1 = \left(\frac{0.03}{0.09} \right)^2 \times 63.64 = 7.07 \text{ m/s}$$

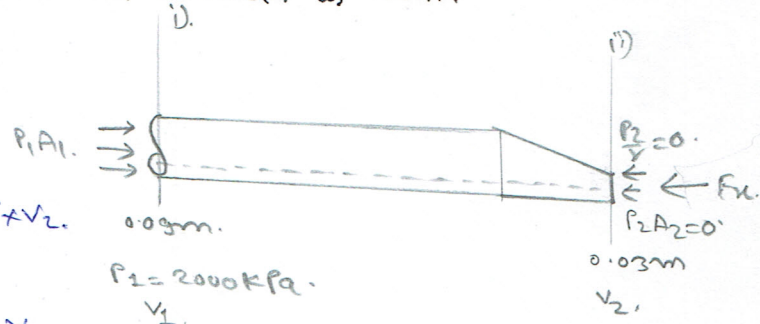
$$Q = A_1 V_1 = \pi \times \frac{0.09^2}{4} \times 7.07 = 0.045 \text{ m}^3/\text{s}$$

Applying momentum equation;

$$\sum F_x = P_1 A_1 - F_x = \rho Q (V_2 - V_1)$$

$$\text{or, } 2000 \times 10^3 \times \pi \times \frac{0.09^2}{4} - F_x = 1000 \times 0.045 (63.64 - 7.07)$$

$$\therefore F_x = 10177.8 \text{ N}$$



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2. water flows in a 10cm diameter pipe at a pressure of 400 kPa out a straight nozzle. calculate the force of water on the nozzle if the outlet diameter is a) 8cm; b) 6cm; c) 4cm. d) 2cm.

Solution;

we have;

continuity equation; $A_i v_i = A_o v_o \Rightarrow v_i = \left(\frac{d_o}{d_i}\right)^2 v_o$; $d_i = 0.1m$

Applying Bernoulli equation at (i) and (o).

$$z_i + \frac{P_i}{\gamma} + \frac{v_i^2}{2g} = z_o + \frac{P_o}{\gamma} + \frac{v_o^2}{2g}$$

$$\frac{400 \times 10^3}{9810} + \frac{v_i^2}{2g} = \frac{v_o^2}{2g}$$

$$v_o^2 - v_i^2 = 800$$

$$\Rightarrow v_o = \sqrt{\frac{800}{\left(1 - \left(\frac{d_o}{d_i}\right)^4\right)}}$$

a) $d_o = 0.08m \Rightarrow v_o = 36.8105 \text{ m/s}$

$$v_i = \left(\frac{0.08}{0.1}\right)^2 \times 36.8105$$

$$= 23.56 \text{ m/s}$$

$$Q = A_i v_i = \pi \times \frac{0.1^2}{4} \times 23.56$$

$$= 0.185 \text{ m}^3/\text{s}$$

$$\sum F_x = P A_i - F_x = \rho Q (v_{o,x} - v_{i,x})$$

$$400 \times 10^3 \times \pi \times \frac{0.1^2}{4} - F_x = 1000 \times 0.185 (36.8105 - 23.56)$$

$$\therefore F_x = 683.25 \text{ N}$$

b) $d_o = 6 \text{ cm} = 0.06 \text{ m}$

$$v_o = 30.317 \text{ m/s}$$

$$v_i = \left(\frac{0.06}{0.1}\right)^2 \times 30.317 = 10.914 \text{ m/s}$$

$$Q = A_i v_i = \pi \times \frac{0.1^2}{4} \times 10.914 = 0.08572 \text{ m}^3/\text{s}$$

$$\sum F_x = P_i A_i - F_x = \rho Q (v_{o,x} - v_{i,x}) \Rightarrow F_x = 1478.367 \text{ N}$$

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$$c) d_0 = 4\text{cm} = 0.04\text{m}.$$

$$v_0 = \sqrt{\frac{800}{1 - \left(\frac{0.04}{0.1}\right)^4}} = 28.65\text{m/s}.$$

$$v_i = \left(\frac{0.04}{0.1}\right)^2 \times 28.65 = 4.6\text{m/s}.$$

$$Q = A_i v_i = \pi \times \frac{0.1^2}{4} \times 4.6 = 0.03613\text{m}^3/\text{s}.$$

$$\sum F_x = P_i A_i - F_x = \rho Q (v_{0x} - v_{ix}).$$

$$\text{or, } 400 \times 10^3 \times \pi \times \frac{0.1^2}{4} - F_x = 1000 \times 0.03613 (28.65 - 4.6)$$

$$\therefore F_x = 2272.66\text{N}.$$

$$d) d_0 = 2\text{cm} = 0.02\text{m}.$$

$$v_0 = \sqrt{\frac{800}{1 - \left(\frac{0.02}{0.1}\right)^4}} = 28.307\text{m/s}.$$

$$v_i = \left(\frac{0.02}{0.1}\right)^2 \times 28.307 = 1.132\text{m/s}.$$

$$Q = A_i v_i = \pi \times \frac{0.1^2}{4} \times 1.132 = 0.008892\text{m}^3/\text{s}.$$

$$\sum F_x = P_i A_i - F_x = \rho Q (v_{0x} - v_{ix}).$$

$$\text{or, } 400 \times 10^3 \times \pi \times \frac{0.1^2}{4} - F_x = 1000 \times 0.008892 (28.307 - 1.132).$$

$$\therefore F_x = 2899.95\text{N}.$$

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3. Find the horizontal and vertical force components of the water on the horizontal bend as shown in figure below if the $P_1 = (a) 200 \text{ kPa}$ $b) 400 \text{ kPa}$ and $c) 800 \text{ kPa}$.

Solution;

a) $P_1 = 200 \text{ kPa}$, $d_1 = 0.08 \text{ m}$, $d_2 = 0.04 \text{ m}$.

Continuity equation;

$$A_1 V_1 = A_2 V_2 \Rightarrow V_1 = \left(\frac{d_2}{d_1} \right)^2 V_2$$

Bernoulli equation at 1) and 2),

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{200 \times 10^3}{9810} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$\therefore V_2^2 - V_1^2 = 400$$

$$V_2^2 \left(1 - \left(\frac{d_2}{d_1} \right)^4 \right) = 400$$

$$\therefore V_2 = \frac{\sqrt{400}}{\sqrt{1 - \left(\frac{d_2}{d_1} \right)^4}} = 20.656 \text{ m/s}$$

$$V_1 = \left(\frac{0.08}{0.04} \right)^2 \times 20.656 = 5.164 \text{ m/s}$$

$$Q = A_1 V_1 = \pi \times \frac{0.08^2}{4} \times 5.164 = 0.026 \text{ m}^3/\text{s}$$

$$\sum F_x = P_1 A_1 - F_x = \rho Q (V_{2x} - V_{1x})$$

$$\text{or, } 200 \times 10^3 \times \pi \times \frac{0.08^2}{4} - F_x = 1000 \times 0.026 (0 - 5.164)$$

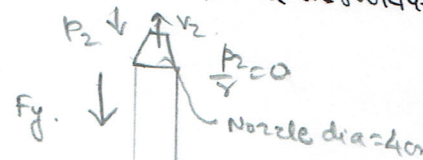
$$\therefore F_x = 133.57 \text{ N} (\leftarrow)$$

$$\sum F_y = -F_y = \rho Q (V_{2y} - V_{1y})$$

$$= 1000 \times 0.026 (20.656 - 0)$$

$$\therefore F_y = -537.056 \text{ N}$$

$$\therefore F_y = 537.056 \text{ N} (\uparrow)$$



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5. A 0.6m pipe carrying 0.9 m³/sec of oil of $S = 0.85$ has a 90° bend in horizontal plane. The loss of head in the bend is 1.0m of air and the pressure at entrance is 3 bars. Determine the resultant force exerted by the oil on the bend.

Solution;

$$V_1 = \frac{0.9}{\pi \times 0.6^2} \times 4 = 3.183 \approx 3.2 \text{ m/s.}$$

$$H_1 = H_2 + h_L.$$

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + H_L.$$

$$\frac{3 \times 10^5}{9810 \times 0.85} + \frac{3.2^2}{2 \times 9.81} = \frac{P_2}{\gamma} + \frac{3.2^2}{2 \times 9.81} + 1.$$

$$\therefore P_2 = 291661.5 \text{ N/m}^2.$$

$$P_1 = P_1 \times A_1 = 3 \times 10^5 \times \pi \times \frac{0.6^2}{4} = 84823.00 \text{ N.}$$

$$P_2 = P_2 \times A_2 = 291661.5 \times \pi \times \frac{0.6^2}{4} = 82465.34 \text{ N.}$$

Momentum Principle;

$$\sum F_x = P_1 + F_x = \rho Q (V_{2x} - V_{1x}).$$

$$\text{or, } 84823 + F_x = 1000 \times 0.9 \times (0 - 3.2) \times 0.85.$$

$$\therefore F_x = 87271 \text{ N.}$$

$$\sum F_y = -P_2 + F_y = \rho Q (V_{2y} - V_{1y})$$

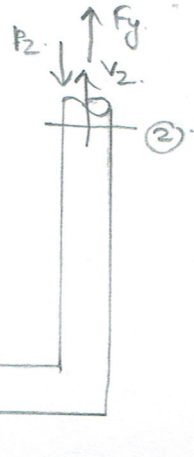
$$\text{or, } -82465.34 + F_y = 1000 \times 0.85 \times 0.9 (3.2 - 0)$$

$$\therefore F_y = 84913.34 \text{ N.}$$

$$\therefore F = \sqrt{F_x^2 + F_y^2} = \sqrt{87271^2 + 84913.34^2}$$

$$= 121764.13 \text{ N.}$$

$$= 121.764 \text{ KN.}$$



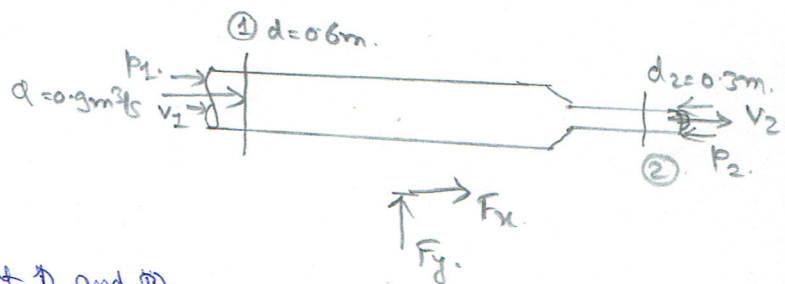
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6. A 0.6m pipe carrying $0.9 \text{ m}^3/\text{s}$ of oil of $S = 0.85$ is connected to a 0.3m pipe by standard reducer fitting. For the entrance pressure of 2.75 bar, what force is exerted by the oil of reducer, neglect any loss of head.

Solution;

$$V_1 = \frac{0.9}{\pi \times 0.6^2} \times 4 = 3.2 \text{ m/s}$$

$$V_2 = \frac{0.9}{\pi \times 0.3^2} \times 4 = 12.8 \text{ m/s}$$



Applying Bernoulli equation at 1 and 2.

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\text{or } \frac{2.75 \times 10^5}{9810 \times 0.85} + \frac{3.2^2}{2 \times 9.81} = \frac{P_2}{\gamma} + \frac{12.8^2}{2 \times 9.81}$$

$$\therefore P_2 = 209720 \text{ N/m}^2$$

Applying momentum principle;

$$\sum F_x = P_1 A_1 - P_2 A_2 + F_x = \rho Q (V_2 - V_1)$$

$$\text{or } 2.75 \times 10^5 \times \pi \times \frac{0.6^2}{4} - 209720 \times \pi \times \frac{0.3^2}{4} + F_x = 1000 \times 0.85 \times 0.9 \times (12.8 - 3.2)$$

$$\therefore F_x = -55586.2 \text{ N} = -55.6 \text{ kN}$$

$$F_y = 0$$

$$\therefore F = F_x = -55.56 \text{ kN}$$

7. A pipe of 30cm diameter carrying $0.3 \text{ m}^3/\text{s}$ of water has a right angled bend in a horizontal plane. Find the resultant force exerted on the bend if the pressure at inlet and the bend are 2.5 kgf/cm^2 and 2.4 kgf/cm^2 , respectively.

Solution;

$$V_1 = \frac{0.3}{\pi \times 0.3^2} \times 4 = 4.24 \text{ m/s}$$

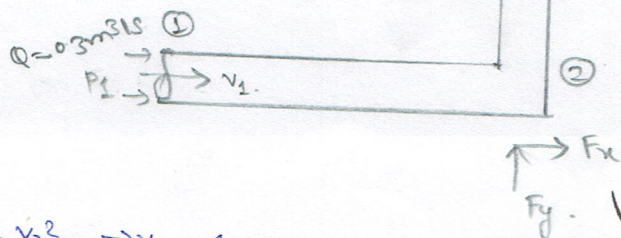
$$A_2 V_2 = Q_2 = Q_1$$

$$A_2 = \frac{Q}{V_2}$$

Bernoulli equation 1 and 2.

$$z_1 + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\text{or } \frac{2.5 \times 250}{9810} + \frac{4.24^2}{2 \times 9.81} = \frac{2.4 \times 40}{9810} + \frac{V_2^2}{2g} \Rightarrow V_2 = 6.13 \text{ m/s} (\uparrow)$$



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$$\therefore A_2 = \frac{0.3}{6.12} = 0.049 \text{ m}^2$$

Momentum principle;

$$\sum F_x = P_1 x A_1 + F_x = \rho Q (v_{2x} - v_{1x})$$

$$\text{or, } 245250 \times 7 \times \frac{0.3^2}{4} + F_x = 1000 \times 0.3 \times (0 - 4.24)$$

$$\therefore F_x = -18607.7 \text{ N} = -1896.81 \text{ kgf}$$

8.

A reducing bend is incorporated in a pipeline so that the direction of flow is turned through 60° in the horizontal plane and the pipe diameter is reduced from 0.25 m to 0.15 m . The velocity and the pressure at the entry to bend are 1.5 m/s and 300 kN/m^2 gauge respectively and the exit. The pressure is 287.2 kN/m^2 . Find the magnitude and direction of the reaction force in the bend in the horizontal plane due to the flowing water.

Solution;

$$Q = A_1 v_1 = 7 \times \frac{0.25^2}{4} \times 1.5$$

$$= 0.0736 \text{ m}^3/\text{s}$$

$$v_2 = \frac{0.0736}{7 \times 0.15^2} \times 4 = 4.165 \text{ m/s}$$

$$P_1 = P_1 \times A_1 = 300 \times 10^3 \times 7 \times \frac{0.25^2}{4} = 14726.21 \text{ N}$$

$$P_2 = P_2 \times A_2 = 287.2 \times 10^3 \times 7 \times \frac{0.15^2}{4} = 5075.243 \text{ N}$$

Momentum principle;

$$\sum F_x = P_1 x - P_2 x + F_x = \rho Q (v_{2x} - v_{1x})$$

$$\text{or, } 14726.21 - 2537.6215 + F_x = 1000 \times 0.0736 \times (2.0825 - 1.5)$$

$$\therefore F_x = -12145.763 \text{ N}$$

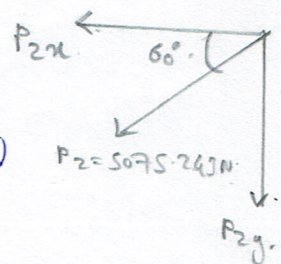
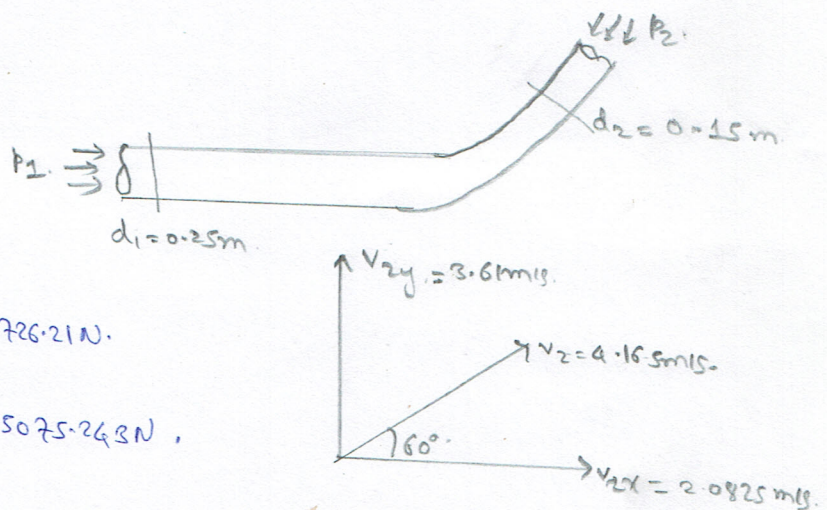
$$\sum F_y = -P_2 y + F_y = \rho Q (v_{2y} - v_{1y})$$

$$\text{or, } -4395.29 + F_y = 1000 \times 0.0736 \times (3.61 - 0)$$

$$\therefore F_y = 4660.986 \text{ N}$$

$$\therefore F = \sqrt{F_x^2 + F_y^2} = 131 \text{ N}$$

$$\therefore \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = 21^\circ$$



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9. A 30° bend reduces in diameter from 15cm to 7.5cm from upper section 1 to lower section 2. water flows through the bend at the rate of 50 lps under a pressure of 1.75 kg/cm^2 measured at section 1. neglecting bend loss find the force exerted by water on the bend and its direction.

solution;

$$v_2 = \frac{0.05}{\pi \times 0.075^2} \times 4 = 11.32 \text{ m/s.}$$

$$v_1 = \frac{0.05}{\pi \times 0.15^2} \times 4 = 2.83 \text{ m/s}$$

$$P_1 = P_1 \times A_1 = 1.75 \times 7 \times 0.15^2 \times \frac{4}{\pi}$$

$$= 3033.74 \text{ N.}$$

Bernoulli equation between 1 and 2.

$$z_1 + \frac{P_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{P_2}{\gamma} + \frac{v_2^2}{2g}$$

$$\text{or } \frac{1.75 \times 7}{9.81} + \frac{2.83^2}{2 \times 9.81} = \frac{P_2}{\gamma} + \frac{11.32^2}{2 \times 9.81}$$

$$\therefore P_2 = 111608.25 \text{ N/m}^2$$

$$P_2 = P_2 \times A_2 = 111608.25 \times \pi \times \frac{0.075^2}{4} = 493.07 \text{ N.}$$

Momentum principle;

$$\sum F_x = P_{2x} - P_{1x} + F_x = \rho Q (v_{2x} - v_{1x})$$

$$\text{or } 493.07 - 2627.3 + F_x = 1000 \times 0.05 \times (-11.32 + 2.45)$$

$$\therefore F_x = 1690.73 \text{ N}$$

$$\sum F_y = -P_{2y} + F_y = \rho Q (v_{2y} - v_{1y})$$

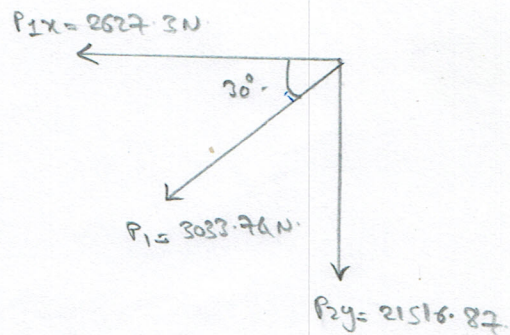
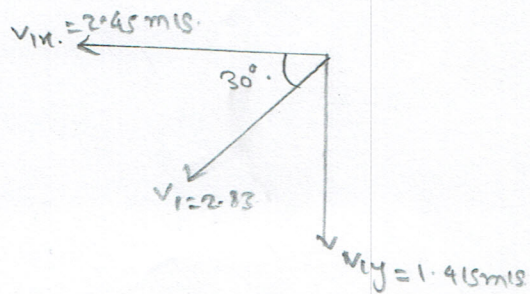
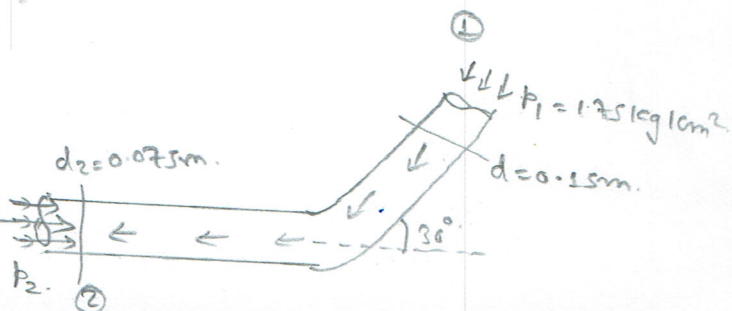
$$\text{or } -1516.87 + F_y = 1000 \times 0.05 (1.415)$$

$$\therefore F_y = 1587.62 \text{ N.}$$

$$\therefore F = \sqrt{F_x^2 + F_y^2} = 2319.29 \text{ N.}$$

$$= 236.42 \text{ kg.}$$

$$\therefore \theta = \tan^{-1} \left(\frac{F_y}{F_x} \right) = 43.2^\circ$$



311bha

11. A horizontal 10cm diameter jet of water with mass 300kg/s strikes a vertical plate. calculate a) the force needed to hold the plate stationary.
b) the force needed to move the plate away from the jet at 10m/s.
c) the force needed to move the plate into the jet at 10m/s.

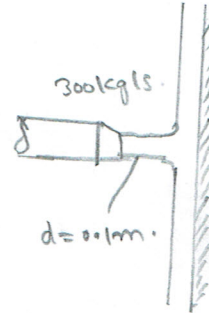
Solution;

a) Plate is stationary.

$$F = \rho Q V = \rho Q \cdot \frac{Q}{A} = \frac{\rho Q^2}{A}$$

$$Q = \frac{m}{\rho t} = \frac{300}{1000} = 0.3$$

$$\therefore F = \frac{1000 \times 0.3^2 \times 4}{\pi \times 0.1^2} = 11459.15 \text{ N.}$$



b) Plate moved away from jet at 10m/s

relative velocity of jet = $V - U$; $V = \frac{Q}{A} = \frac{0.3}{\pi \times 0.1^2} \times 4 = 38.2 \text{ m/s}$

$$F = \rho A (V - U)^2 = 1000 \times \pi \times 0.1^2 \times (38.2 - 10)^2 = 6245.8 \text{ N}$$

c) Plate moves into the jet at 10m/s

relative velocity of jet wrt. plate = $V + U$; $V = 38.2 \text{ m/s}$
 $U = 10 \text{ m/s}$

$$F = \rho A (V + U)^2 = 1000 \times \pi \times 0.1^2 \times (38.2 + 10)^2 = 18246.58 \text{ N}$$

12. Find the force exerted by a jet of water of diameter 7.5cm on a stationary flat plate, when the jet strikes the plate normally with a velocity of 20m/s.

Solution;

$$d = 7.5 \text{ cm} = 0.075 \text{ m}$$

$$V = 20 \text{ m/s}$$

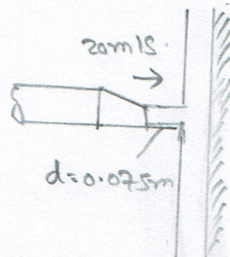
Plate is stationary.

$$F = \rho A V^2$$

$$= \rho \times \pi \times \frac{d^2}{4} \times V^2$$

$$= 1000 \times \pi \times \frac{0.075^2}{4} \times 20^2$$

$$= 1767.14 \text{ N}$$



311bha

13. A jet of water of diameter 7.5 cm moving with a velocity of 25 m/s strikes a fixed plate in such a way that the angle between the jet and plate is 60° . Find the force exerted by the jet on the plate.

- a) In direction normal to the plate and
b) In direction of jet.

Solution;

a) $F_n = \rho A V^2 \sin \theta$

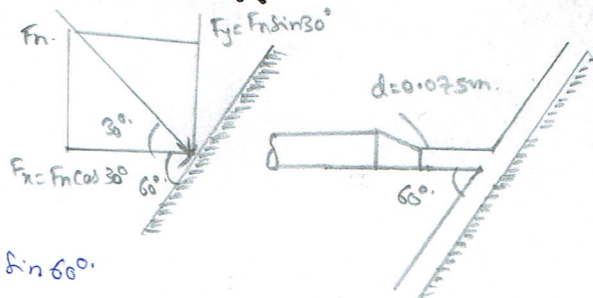
$$= 1000 \times \pi \times \frac{0.075^2}{4} \times 25^2 \times \sin 60^\circ$$

$$= 2391.24 \text{ N}$$

b) $F_n = F_n \cos 30^\circ$

$$= 2391.24 \times \cos 30^\circ$$

$$= 2070.87 \text{ N}$$



14. A jet of water of diameter 5 cm moving with a velocity of 40 m/s strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.

Solution;

We know;

$$F_n = \rho A V^2 (1 + \cos \theta)$$

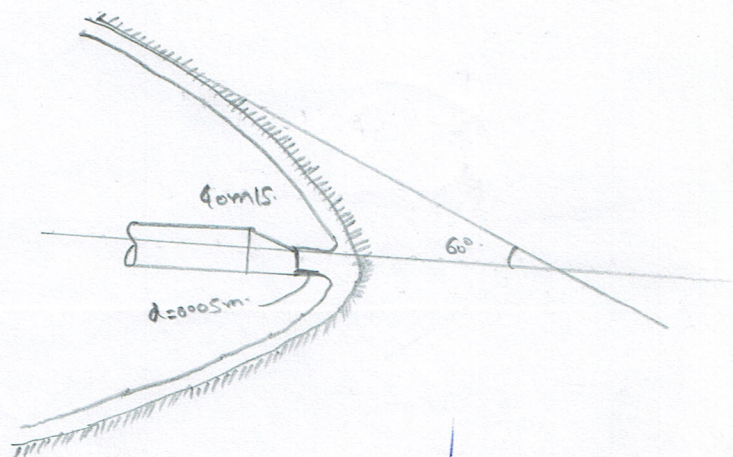
$$\theta = 60^\circ$$

$$\therefore F_n = \rho A V^2 (1 + \frac{1}{2})$$

$$= 1000 \times \pi \times \frac{0.05^2}{4} \times 40^2 \times 1.5$$

$$= 4712.388 \text{ N}$$

$$= 480.36 \text{ kgf}$$

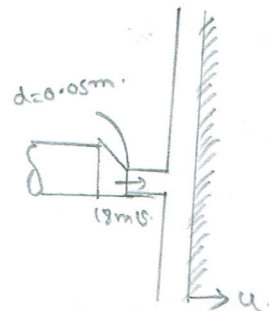


Zillbhar

25. A flat plate is struck normally by a jet of water 50mm in diameter with a velocity of 18 m/s. Calculate (a) the force on the plate when it is stationary (b) the force on the plate when it moved in the same direction as the jet with a velocity of 6 m/s. (c) the work done for second and the efficiency in case b).

Solution;

$$\begin{aligned} \text{a) } F_n &= \rho A V^2 = 1000 \times \pi \times \frac{0.05^2}{4} \times 18^2 \\ &= 636.17 \text{ N} \end{aligned}$$



- b) Plate moved in the direction of jet,
Relative velocity of jet wrt plate is;

$$v - u = 18 - 6 = 12 \text{ m/s.}$$

$$\therefore F_n = \rho A (v - u)^2 = 1000 \times \pi \times \frac{0.05^2}{4} \times 12^2 = 282.74 \text{ N.}$$

c) Efficiency.

$$\text{Initial energy per unit time} = \frac{1}{2} \frac{mv^2}{t} = \frac{1}{2} \times \frac{V \times \rho}{t} \times V^2$$

$$= \frac{1}{2} \rho V^3 = \frac{1}{2} \rho A V^3$$

$$\text{Work done by plate} = \rho A (v - u)^2 \times u.$$

$$\therefore \frac{1}{\eta} = \frac{\frac{1}{2} \rho A V^3}{\rho A (v - u)^2 \times u} = \frac{\frac{1}{2} \times 18^3}{12^2 \times 6} \Rightarrow \eta = \frac{12^2 \times 6}{\frac{1}{2} \times 18^3} \times 100\%$$

$$= 29.63\%$$

Zillbhar

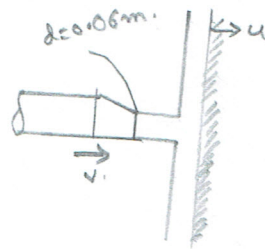
17. A horizontal 5cm diameter jet of water strikes a vertical plate. Determine the velocity issuing from the jet if a force of 900N is needed for a) hold the plate stationary b) move the plate away from the jet at 10m/s c) move the plate into the jet at 10m/s.

Solution;

a) plate is stationary, $u=0$.

$$F = \rho A v^2 \Rightarrow 900 = 1000 \times \pi \times \frac{0.05^2}{4} \times v^2.$$

$$\therefore v = 17.84 \text{ m/s}$$



b) plate moves away from jet i.e. relative velocity of jet wrt. plate = $v-u$, $u=10 \text{ m/s}$.

$$\therefore F = \rho A (v-u)^2 \Rightarrow 900 = 1000 \times \pi \times \frac{0.05^2}{4} (v-10)^2$$

$$\therefore v = 27.84 \text{ m/s}$$

c) plate move into jet with 10m/s i.e. relative velocity of jet wrt. plate = $v-(u) = v+u = v+10$.

$$\therefore F = \rho A (v+u)^2$$

$$\text{or } 900 = 1000 \times \pi \times \frac{0.05^2}{4} \times (v+10)^2$$

$$\therefore v = 7.84 \text{ m/s}$$

20. Calculate the force components of the water acting on the deflecting blade as shown in figure below if a) blade is stationary b) the blade moves to the right at 20m/s and c) the blade move to the left at 20m/s.

Solution;

a) plate is stationary, $u=0$.

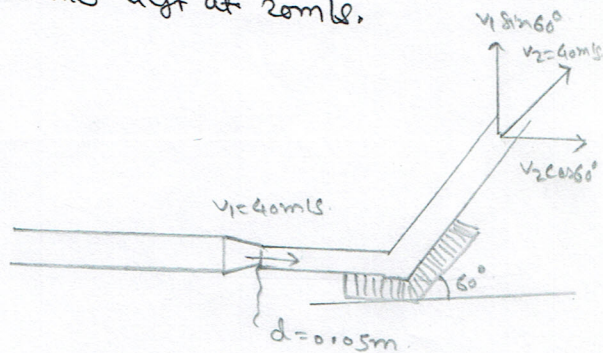
$$\therefore F_x = \rho A v_1 (v_{1x} - v_{2x}); Q = A v_1$$

$$= 1000 \times \pi \times \frac{0.05^2}{4} \times 40 [40 - 40 \cos 60^\circ]$$

$$= 1570.8 \text{ N}$$

$$F_y = \rho A v_1 (v_{1y} - v_{2y}) = 1000 \times \pi \times \frac{0.05^2}{4} \times 40 \times (0 - 40 \sin 60^\circ)$$

$$= -2720.7 \text{ N}$$



311bhar

b) Blade move right at 20m/s then relative velocity of jet w.r.t. blade $= v-u = 40-20$
 $= 20\text{m/s}$.

$$F_x = \rho A (v-u) [(v-u)_{1x} - (v-u)_{2x}]$$

$$= 1000 \times \pi \times \frac{0.05^2}{4} \times 20 [20 - 20 \cos 60^\circ]$$

$$= 392.7 \text{ N}.$$

$$F_y = \rho A (v-u) [(v-u)_{1y} - (v-u)_{2y}]$$

$$= 1000 \times \frac{0.05^2}{4} \times \pi \times 20 [0 - 20 \sin 60^\circ]$$

$$= -680.17 \text{ N}.$$

c) Blade moves to the left at 20m/s then relative velocity of jet w.r.t. blade
 $= v - (-u) = v+u = 40+20 = 60\text{m/s}$.

$$F_x = \rho A (v+u) [(v+u)_{1x} - (v+u)_{2x}]$$

$$= 1000 \times \pi \times \frac{0.05^2}{4} \times 60 (60 - 60 \cos 60^\circ)$$

$$= 3534.3 \text{ N}.$$

$$F_y = \rho A (v+u) [(v+u)_{1y} - (v+u)_{2y}]$$

$$= 1000 \times \pi \times \frac{0.05^2}{4} \times 60 (0 - 60 \sin 60^\circ)$$

$$= -6121.57 \text{ N}.$$

Zillbhar

21.

Calculate the force components of the water acting on the deflecting blade as shown in figure below if a) blade is stationary b) the blade moves to the right at 100 m/s and c) the blade moves to the left at 100 m/s.

Solution;

a) Blade is stationary

$$F_x = \rho A V_1 [V_{1x} - V_{2x}]$$

$$= 1000 \times \pi \times \frac{0.04^2}{4} \times 40 \times (40 + 40 \cos 60^\circ)$$

$$= 2015.928 \text{ N}$$

$$F_y = \rho A V_1 [V_{1y} - V_{2y}]$$

$$= 1000 \times \pi \times \frac{0.04^2}{4} \times 40 \times (0 - 40 \sin 60^\circ)$$

$$= -1741.247 \text{ N}$$

b) Blade moves to the right at 100 m/s i.e. relative velocity of jet wrt blade $= v - u = 40 - 100 = -60 \text{ m/s}$.

$$F_x = \rho A (v - u) [(v - u)_x - (v - u)_x]$$

$$= 1000 \times \pi \times \frac{0.04^2}{4} \times (-60) [-60 - (-(-60 \cos 60^\circ))] = 678.584 \text{ N}$$

$$F_y = \rho A (v - u) [(v - u)_y - (v - u)_y]$$

$$= 1000 \times \pi \times \frac{0.04^2}{4} \times (-60) (0 - (-60 \sin 60^\circ))$$

$$= 3917.8 \text{ N}$$

c) Blade moves to the left at 100 m/s i.e. relative velocity of jet wrt blade $= v - (-u) = v + u = 40 + 100 = 140 \text{ m/s}$

$$F_x = \rho A (v + u) [(v + u)_x - (v + u)_x]$$

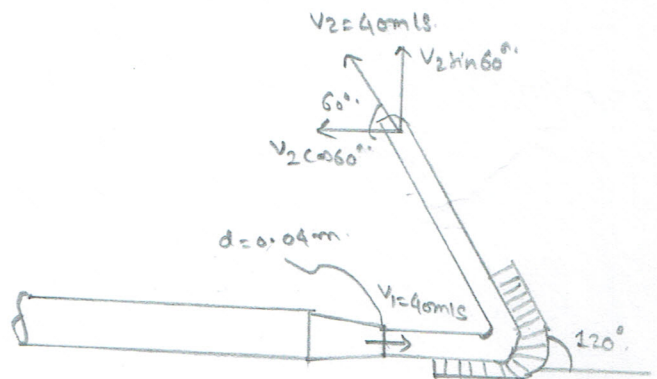
$$= 1000 \times \pi \times \frac{0.04^2}{4} \times 140 [140 + 140 \cos 60^\circ]$$

$$= 36945.12 \text{ N}$$

$$F_y = \rho A (v + u) [(v + u)_y - (v + u)_y]$$

$$= 1000 \times \pi \times \frac{0.04^2}{4} \times 140 [0 - 140 \sin 60^\circ]$$

$$= -21330.28 \text{ N}$$



311bhar

22.

water flows at 15 m/s in a 10 cm diameter stem of a horizontal T-section that branches into 5 cm diameter pipes. Find the force of water on the T-section if the branches exit to the atmosphere. Neglect viscous effect.

Solution;

$$Q = \frac{\pi \times 0.1^2}{4} \times 15 = 0.1178 \text{ m}^3/\text{s}.$$

$$v_2 = \frac{Q_2}{A_2} = \frac{0.1178}{2 \times \frac{\pi \times 0.05^2}{4}} \times 4.$$

$$= 29.99 \approx 30 \text{ m/s} = v_3.$$

Applying Bernoulli Equation between ① and ②,

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g}$$

$$\text{or, } \frac{p_1}{\gamma} + \frac{15^2}{2 \times 9.81} = 0 + 0 + \frac{30^2}{2 \times 9.81}.$$

$$\Rightarrow p_1 = 337500 \text{ N/m}^2$$

Applying momentum equation;

$$\sum F_x = p_1 \times A_1 + F_x.$$

$$= \rho \frac{Q}{2} (v_2 x) + \rho \frac{Q}{2} (v_3 x) - \rho Q v_{1x} = -\rho Q v_{1x}.$$

$$\text{or, } 337500 \times \frac{\pi \times 0.1^2}{4} + F_x = -1000 \times 0.1178 (15).$$

$$\therefore F_x = -4417.72 \text{ N}.$$

$$\sum F_y = -p_2 \times A_1 + p_3 \times A_3 + F_y$$

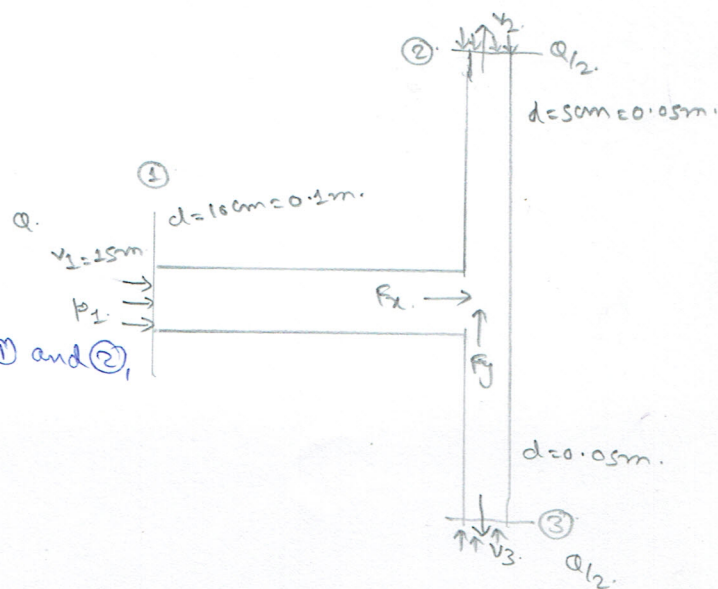
$$= \rho \frac{Q}{2} (v_{3y} + v_{2y}) - \rho Q v_{1y}.$$

$$0 + 0 + F_y = \rho \frac{Q}{2} (30 - 30) - \rho Q \times 0.$$

$$\therefore F_y = 0.$$

$$\therefore F = \sqrt{F_x^2 + F_y^2} = \sqrt{F_x^2 + 0} = F_x.$$

$$\therefore F = F_x = 4417.72 \text{ N}.$$



311bhar

- 23 water flows from the rectangular jet as shown in figure below. Find the forces and mass fluxes m_2 and m_3 if the deflection angle is;
 a) 60° b) 45° c) 30° .

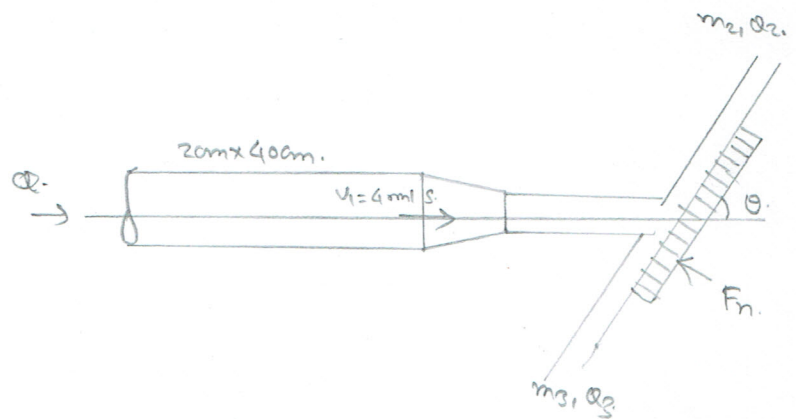
Solution;

$$F_n = \rho A v^2 \sin \theta$$

$$Q_2 = \frac{Q}{2} (1 + \cos \theta)$$

$$\Rightarrow m_2 = \frac{\rho Q}{2} (1 + \cos \theta)$$

$$m_3 = \frac{\rho Q}{2} (1 - \cos \theta)$$



a) $\theta = 60^\circ$.

$$F_n = 1000 \times (0.02 \times 0.4) \times 40^2 \times \sin 60^\circ$$

$$= 11085.125 \text{ N}$$

$$m_2 = 1000 \times \frac{0.02 \times 0.4 \times 40}{2} \times (1 + \cos 60^\circ)$$

$$= 240 \text{ kg/s}$$

$$m_3 = 1000 \times \frac{0.02 \times 0.4 \times 40}{2} \times (1 - \cos 60^\circ)$$

$$= 80 \text{ kg/s}$$

b) $\theta = 45^\circ$.

$$F_n = 1000 \times 0.02 \times 0.4 \times 40^2 \times \sin 45^\circ$$

$$= 9050.96 \text{ N}$$

$$m_2 = \frac{1000 \times 0.02 \times 0.4 \times 40}{2} \times (1 + \cos 45^\circ)$$

$$= 273.13 \text{ kg/s}$$

$$m_3 = 1000 \times \frac{0.02 \times 0.4 \times 40}{2} \times (1 - \cos 45^\circ)$$

$$= 46.86 \text{ kg/s}$$

c) $\theta = 30^\circ$.

$$F_n = 1000 \times 0.02 \times 0.4 \times 40^2 \times \sin 30^\circ = 6400 \text{ N}$$

$$m_2 = 1000 \times \frac{0.02 \times 0.4 \times 40}{2} \times (1 + \cos 30^\circ) = 298.65 \text{ kg/s}$$

$$m_3 = 1000 \times \frac{0.02 \times 0.4 \times 40}{2} \times (1 - \cos 30^\circ) = 21.43 \text{ kg/s}$$

311bhan

24. A 150 mm diameter jet moving at 30 m/s impinges on a series of vanes moving at 15 m/s in the direction of jet. The jet leaves the vanes at 60° with the direction of motion of the vanes as shown in figure below. Calculate. a) the force exerted by the jet in the direction of motion of the vanes, b) work done by the jet per second.

Solution;

$$a) F_x = \rho Q (V_{1x} - V_{2x})$$

$$V_{1x} = V - u = 30 - 15 = 15 \text{ m/s}$$

$$V_{2x} = -(V - u) \cos 60^\circ = -15 \cos 60^\circ$$

$$= -\frac{15}{2} = -7.5 \text{ m/s}$$

For series of vane; $Q = AV = \frac{\pi}{4} \times 0.15^2 \times 30$

$$= 0.53 \text{ m}^3/\text{s}$$

$$\therefore F_x = 1000 \times 0.53 \times (15 - (-7.5))$$

$$= 11925 \text{ N}$$

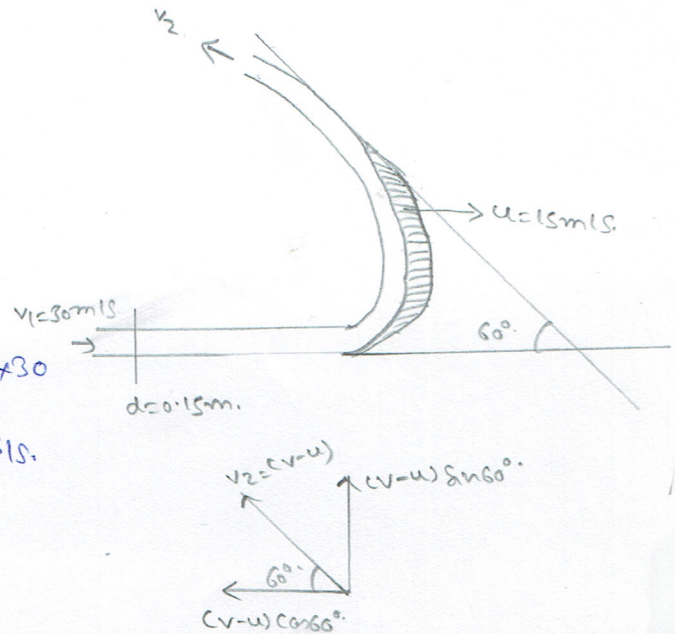
b) work done by the jet per second

$$= F_x \times u$$

$$= 11925 \times 15$$

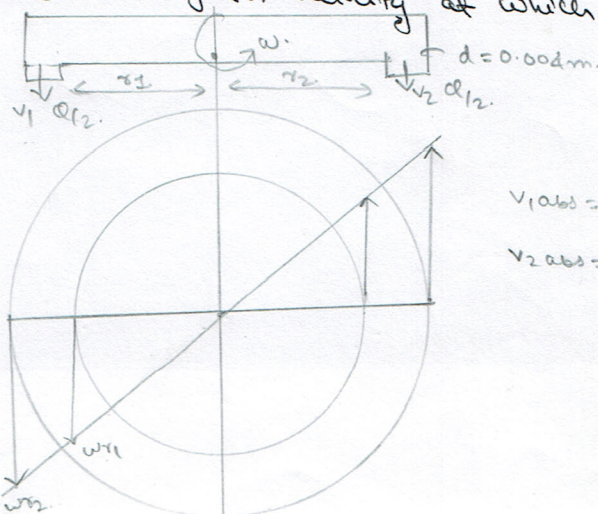
$$= 178875 \text{ Nm/s}$$

$$= 178.875 \text{ kNm/Sec}$$



27. A lawn sprinkler with two nozzles of diameter 4 mm each is connected across a tap of water as shown in figure below. The rate of flow through tap is $120 \text{ cm}^3/\text{s}$. The nozzles discharge water in downstream direction. Determine the angular velocity at which the sprinkler will rotate free.

Solution;



$$V_{1 \text{ abs}} = V_1 + \omega r_1$$

$$V_{2 \text{ abs}} = V_2 - \omega r_2$$

311bha

$$\frac{Q}{2} = \frac{7 \times 0.004^2}{4} \times V.$$

$$\Rightarrow V = \frac{120 \times 10^{-6}}{2 \times 7 \times 0.004^2} \times 4$$

$$= 4.77 \text{ m/s.}$$

Sprinkler is free to rotate.

$$\therefore T_1 = T_2.$$

$$S_2 (V - \omega r_2) r_2 = S_1 (V + \omega r_1) r_1$$

$$(4.77 - \omega \times 0.3) \times 0.3 = (4.77 + \omega \times 0.2) \times 0.2$$

$$4.77 \times 0.3 - \omega \times 0.3^2 = 4.77 \times 0.2 + \omega \times 0.2^2$$

$$\therefore \omega = 3.67 \text{ rad/sec.}$$

28.

A lawn sprinkler has two arms each containing a nozzle of 0.02 cm^2 area at a radius 15 cm from the rotor axis as shown in fig. below. The nozzles are tangentially apposed and discharge horizontally at an angle of 70° with the radius. If the rate of flow is 0.72 lit/min find a) the torque on the rotor which is produced when it is held stationary b) the speed of rotor if no friction or resistance (no external energy) or rotate freely.

Solution:

$$\text{Area of nozzle} = 0.02 \text{ cm}^2.$$

$$= 0.02 \times 10^{-4} \text{ m}^2$$

$$Q = 0.72 \text{ lit/min.}$$

$$= \frac{0.72 \times 10^{-3}}{60}$$

$$= 12 \times 10^{-6} \text{ m}^3/\text{s.}$$

Now,

$$V = \frac{Q}{2A} = \frac{12 \times 10^{-6}}{2 \times 0.02 \times 10^{-4}} = \frac{6}{2} \text{ m/s.}$$

$$= V_A = V_B = 3 \text{ m/s}$$

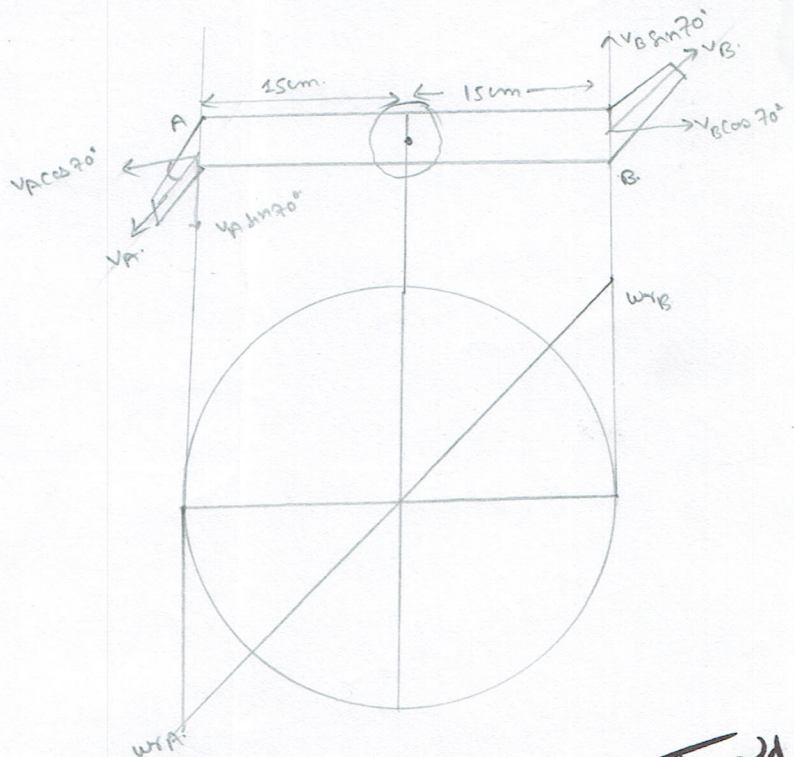
$$a) T = T_A + T_B.$$

$$= 2T_A$$

$$= 2 S_2 \frac{(V_A)^2}{2} r_A$$

$$= 2 \times 1000 \times \frac{12 \times 10^{-6}}{2} \times 3 \sin 70^\circ \times 0.15.$$

$$= 5.074 \times 10^{-3} \text{ Nm.}$$



311bhar

Tutorial 7

Boundary Layer Theory

by
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3. Water at 50°C flows in a 15cm diameter pipe with velocity 6.5 m/s and absolute roughness height 0.14 mm. Head loss measurement indicate that friction factor $f = 0.020$. a) what is the thickness of viscous sub layer? b) Is the pipe behaving as a fully rough pipe?

Solution;

Given;

$$V = 0.556 \times 10^{-6} \text{ m}^2/\text{s}.$$

$$D = 15 \text{ cm} = 0.15 \text{ m}.$$

$$V = 6.5 \text{ m/s}.$$

$$f = 0.020.$$

$$e = 0.14 \text{ mm}.$$

$$a) Re = \frac{D \times V}{V} = \frac{0.15 \times 6.5}{0.556 \times 10^{-6}} = 1.75 \times 10^6.$$

For turbulent flow, $Re > 10^6$ so, it is turbulent flow

$$\therefore \text{Thickness of viscous sub-layer, } \delta' = \frac{14.14 \times V}{V \sqrt{f}}.$$

$$= \frac{14.14 \times 0.556 \times 10^{-6}}{6.5 \times \sqrt{0.020}}.$$

$$= 8.55 \times 10^{-6} \text{ m}.$$

b) For fully rough pipe;

$$e > 14\delta'$$

$$\Rightarrow 0.14 \times 10^{-3} > 14 \times 8.55 \times 10^{-6} \quad (\text{true}).$$

$\therefore$ The pipe is behaving as a fully rough pipe.

4. A smooth two dimensional flat plate is exposed to a wind velocity of 360 km/hr. If the laminar boundary exists upto a value of $Re_x = 2 \times 10^5$, find the maximum distance from the leading edge upto which laminar boundary layer exist and maximum thickness, taking the kinematic viscosity of air $= 1.49 \times 10^{-5} \text{ m}^2/\text{s}$.

Solution;

$$V = \frac{360 \times 10^3}{3600} = 100 \text{ m/s}.$$

$$Re = 2 \times 10^5$$

$$V = 1.49 \times 10^{-5}.$$

$$\text{we have; } Re = \frac{Vx}{V} \Rightarrow x = \frac{Re \cdot V}{V} = \frac{2 \times 10^5 \times 1.49 \times 10^{-5}}{100} = 0.0298 \text{ m} = 29.8 \text{ mm}.$$

311bha

For laminar layer, $\frac{\delta}{x} = \frac{5}{\sqrt{Re}}$

$$\delta = \frac{5 \times 29.8}{\sqrt{2 \times 10^5}}$$

$$= 0.33 \text{ mm.}$$

5. A free-stream flows with a velocity of 1 m/s , density 1000 kg/m^3 and dynamic viscosity is $0.01 \text{ Pa}\cdot\text{s}$ flows over a horizontal flat plate. Calculate the Reynold's number and the boundary layer thickness at 0.1 m from the leading edge.

Solution;

$$V = 1 \text{ m/s}, \rho = 1000 \text{ kg/m}^3, \mu = 0.01 \text{ Pa}\cdot\text{s}, \nu = \frac{\mu}{\rho} = \frac{0.01}{1000} = 10^{-5}$$

$$x = 0.1 \text{ m.}$$

we know;

$$Re = \frac{Vx}{\nu} = \frac{1 \times 0.1}{10^{-5}} = 10^4$$

$$\text{Boundary layer thickness (for laminar). } \frac{\delta}{x} = \frac{5}{\sqrt{Re}}$$

$$\therefore \delta = \frac{5 \times 0.1}{\sqrt{10^4}}$$

$$= 0.005 \text{ m.}$$

6. The laminar boundary layer profile in a case is approximated by a cubic parabola as $\frac{u}{V} = \frac{3}{2} \left(\frac{y}{\delta}\right)^2 - \frac{1}{2} \left(\frac{y}{\delta}\right)^3$, where u is the velocity at a distance y from the surface and $y \rightarrow \delta, u \rightarrow V$. Calculate the displacement thickness and momentum thickness in terms of δ and workout the shear stress at the surface.

Solution;

$$\frac{u}{V} = \frac{3}{2} \left(\frac{y}{\delta}\right)^2 - \frac{1}{2} \left(\frac{y}{\delta}\right)^3$$

$$\text{Displacement thickness, } \delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy.$$

$$= \int_0^\delta \left(1 - \frac{3}{2} \frac{y^2}{\delta^2} + \frac{1}{2} \frac{y^3}{\delta^3}\right) dy.$$

Bibha

$$= \left[y - \frac{3y^3}{6\delta^2} + \frac{1}{2} \frac{y^4}{4\delta^3} \right]_0^{\delta}$$

$$= \delta \left[1 - \frac{1}{2} + \frac{1}{8} \right]$$

$$= \frac{5}{8} \delta$$

$$\text{momentum thickness, } \theta = \int_0^{\delta} \frac{u}{V} \left[1 - \frac{u}{V} \right] dy$$

$$= \int_0^{\delta} \frac{u}{V} - \left(\frac{u}{V} \right)^2 dy$$

$$= \int_0^{\delta} \left[\frac{3}{2} \frac{y^2}{\delta^2} - \frac{1}{2} \frac{y^3}{\delta^3} - \frac{9}{4} \frac{y^4}{\delta^4} - \frac{1}{4} \frac{y^6}{\delta^6} + 2 \cdot \frac{3}{2} \frac{y^5}{\delta^5} \cdot \frac{1}{2} \right] dy$$

$$= \left[\frac{3}{2} \cdot \frac{y^3}{3\delta^2} - \frac{1}{2} \cdot \frac{y^4}{4\delta^3} - \frac{9}{4} \cdot \frac{y^5}{5\delta^4} - \frac{1}{4} \frac{y^7}{7\delta^6} + \frac{3}{2} \frac{y^6}{6\delta^5} \right]_0^{\delta}$$

$$= \delta \left[\frac{1}{2} - \frac{1}{8} - \frac{9}{20} - \frac{1}{28} + \frac{3}{12} \right]$$

$$= \frac{39\delta}{280}$$

$$\text{At Surface, } u \text{ is constant} \Rightarrow \frac{du}{dy} = 0$$

$$\therefore \tau (\text{shear stress}) = 0$$

7. A flat plate is placed longitudinally in a turbulent stream of water flowing at 10 m/s. locate the transition point and calculate the boundary layer thickness just before the transition.

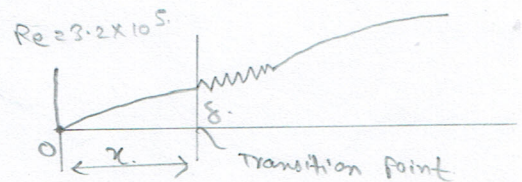
Solution:

$$V = 10 \text{ m/s}, \mu_{H_2O} = \mu = 10^{-6} \text{ m}^2/\text{s}$$

$$Re = \frac{Vx}{\nu} \Rightarrow x = \frac{3.2 \times 10^5 \times 10^{-6}}{10} = 0.032 \text{ m} = 3.2 \text{ cm}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re}}$$

$$\therefore \delta = \frac{5 \times 3.2}{\sqrt{3.2 \times 10^5}} = 0.028 \text{ cm}$$



Bibha

9. Find the expression for the displacement and momentum thickness for the following velocity profile $\frac{u}{V} = \left(\frac{y}{\delta}\right)^{1/m}$ in the boundary layer on a flat plate.

Solution;

Displacement thickness, $\delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy.$

$$= \int_0^\delta \left(1 - \frac{y^{1/m}}{\delta^{1/m}}\right) dy.$$

$$= \left[y - \frac{y^{1/m+1}}{\delta^{1/m}} \times \frac{m}{m+1} \right]_0^\delta.$$

$$= \delta \left[1 - \frac{m}{m+1} \right].$$

$$\therefore \delta^* = \frac{\delta}{m+1}.$$

Momentum thickness, $\theta = \int_0^\delta \frac{u}{V} \left(1 - \frac{u}{V}\right) dy.$

$$= \int_0^\delta \left(\frac{u}{V} - \frac{u^2}{V^2} \right) dy.$$

$$= \int_0^\delta \left(\frac{y^{1/m}}{\delta^{1/m}} - \frac{y^{2/m}}{\delta^{2/m}} \right) dy.$$

$$= \left[\frac{y^{1/m+1}}{\delta^{1/m}} \cdot \frac{m}{1+m} - \frac{y^{2/m+1}}{\delta^{2/m}} \times \frac{m}{m+2} \right]_0^\delta.$$

$$= \delta \left[\frac{m}{m+1} - \frac{m}{m+2} \right].$$

$$= \delta m \left[\frac{1}{(m+1)(m+2)} \right].$$

$$= \frac{\delta m}{(m+1)(m+2)}.$$

Zillbhart

Tutorial 8

Flow Past through Submerged Bodies

by
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1. Calculate the drag force on a smooth 50 cm diameter sphere when subjected to a 20°C atmospheric airflow of a) 6 m/s b) 15 m/s.

Solution;

At 20°C, $\rho = 1.2 \text{ kg/m}^3$, $\mu = 1.8 \times 10^{-5} \text{ Pa.s}$.

$$Re = \frac{\rho V D}{\mu} = \frac{1.2 \times 0.5 \times 6}{1.8 \times 10^{-5}} = 2 \times 10^5$$

$$\therefore C_D = 0.5.$$

$$a) F_D = C_D \times \frac{\rho A V^2}{2} = 0.5 \times 1.2 \times \pi \times \frac{0.5^2}{4} \times \frac{6^2}{2} = 2.12 \text{ N.}$$

b) $V = 15 \text{ m/s}$

$$Re = \frac{1.2 \times 0.5 \times 15}{1.8 \times 10^{-5}} = 5 \times 10^5$$

$$\therefore C_D = 0.5.$$

$$F_D = 0.5 \times 1.2 \times \pi \times \frac{0.5^2}{4} \times \frac{15^2}{2} = 13.25 \text{ N.}$$

2. A 20 cm diameter smooth sphere experiences a drag 2 N when placed in an atmosphere 20°C air. a) what is velocity of air stream? b) At what increased speed will the sphere experience the same drag?

Solution;

$$d = 20 \text{ cm} = 0.2 \text{ m.}$$

$$F_D = 2 \text{ N.}$$

$$\rho = 1.2 \text{ kg/m}^3.$$

$$\mu = 1.8 \times 10^{-5} \text{ Pa.s.}$$

$$\therefore C_D = 0.5.$$

$$\text{or, } F_D = C_D \times \frac{\rho A V^2}{2}$$

$$\text{or, } 2 = 0.5 \times 1.2 \times \pi \times \frac{0.2^2}{4} \times \frac{V^2}{2}$$

$$\therefore V = 14.56 \text{ m/s}$$

311bha

3. A smooth 20 cm diameter sphere experiences drag of 4.2 N when placed in a 20°C water channel. Calculate the drag coefficient and the Reynold's number.

Solution;

$$\text{diameter (d)} = 0.2 \text{ m.}$$

$$F_D = 4.2 \text{ N.}$$

we have;

$$F_D = C_D \times \frac{\rho A V^2}{2}$$

$$\text{or, } 4.2 = C_D \times 1.2 \times \frac{2 \times 10^{-2}}{4} \times \frac{V^2}{2}$$

$$\text{or, } C_D \times V^2 = 222.817$$

$$\text{Also; } Re = \frac{\rho V D}{\mu}$$

$$= \frac{1.2 \times V \times 0.2}{1.8 \times 10^{-5}}$$

$$= 13333.33 V$$

4. A flat plate 1.5 m x 1.5 m moves at 50 km/hr in stationary air of specific weight 1.15 kgf/m³. If the C_D and C_L are 0.15 and 0.75 respectively. Calculate drag force, lift force and resultant force.

Solution;

$$A = 1.5 \times 1.5 = 2.25 \text{ m}^2$$

$$V = 50 \times \frac{5}{18} \text{ m/s} = 13.89 \text{ m/s}$$

$$\gamma = 1.15 \text{ kgf/m}^3$$

$$\therefore \rho = 1.15 \text{ kg/m}^3$$

$$C_D = 0.15 \quad C_L = 0.75$$

we have;

$$F_D = \frac{C_D \rho A V^2}{2} = \frac{0.15 \times 1.15 \times 2.25 \times 13.89^2}{2} \times \frac{1}{9.81}$$

$$= 3.816 \text{ kgf.}$$

$$F_L = \frac{C_L \rho A V^2}{2} = \frac{0.75 \times 1.15 \times 2.25 \times 13.89^2}{2 \times 9.81}$$

$$= 19.083 \text{ kgf}$$

$$\therefore F = \sqrt{F_D^2 + F_L^2} = 19.46 \text{ kgf.}$$

311bha

5. A circular disc 3m in diameter is held normal to a 26.4 m/s and wind density is 0.0012 g/mlcm^3 . What force is required to hold it at rest? Assume coefficient of drag of disc = 1.12.

Solution;

$$d = 3 \text{ m}$$

$$v = 26.4 \text{ m/s}$$

$$\rho = 0.0012 \text{ g/mlcm}^3$$

$$= 0.0012 \times 10^6 \text{ kg/m}^3$$

$$C_D = 1.12$$

we have;

$$F_D = \frac{C_D \rho A v^2}{2}$$

$$= \frac{1.12 \times 1.2 \times \frac{\pi \times 3^2}{4} \times \frac{26.4^2}{2} \times \frac{1}{9.81}}$$

$$= 337.47 \text{ kgf.}$$

6. Air blows over a cylindrical smoke-stack of 5cm diameter with a velocity of 0.2 m/s. Find out the total aerodynamic drag, shear drag and pressure drag on 1m height of the smoke stack. Take total and shear drag coefficients 1.25 and 0.18 respectively and $\rho_{\text{air}} = 1.2 \text{ kg/m}^3$.

Solution;

$$\text{diameter (d)} = 5 \text{ cm} = 0.05 \text{ m}$$

$$v = 0.2 \text{ m/s}$$

$$h = 1 \text{ m}$$

$$\rho_{\text{air}} = 1.2 \text{ kg/m}^3$$

$$C_D = 1.25 \text{ (total)}$$

$$C_D = 0.18 \text{ (shear drag)}$$

Now;

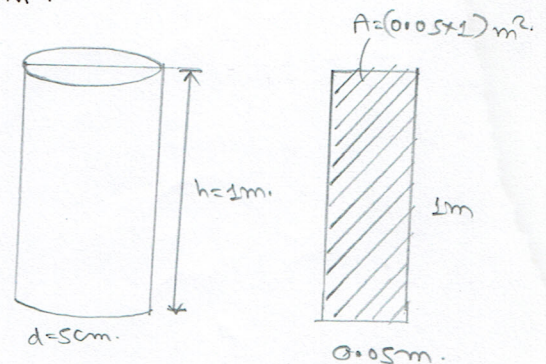
$$\text{Total drag} = \frac{C_D \rho A v^2}{2} = \frac{1.25 \times 1.2 \times (0.05 \times 1) \times 0.2^2}{2} = 1.5 \times 10^{-3} \text{ N}$$

$$\text{Shear drag} = \frac{C_D \rho A v^2}{2} = \frac{0.18 \times 1.2 \times (0.05 \times 1) \times 0.2^2}{2} = 2.16 \times 10^{-4} \text{ N}$$

$$\text{Pressure drag} = \text{Total drag} - \text{Shear drag}$$

$$= 1.5 \times 10^{-3} - 2.16 \times 10^{-4} = 1.284 \times 10^{-4} \text{ N}$$

$$\text{Pressure drag} = \frac{C_D \rho A v^2}{2} \quad \text{or, } 1.284 \times 10^{-4} = \frac{C_D \times 1.2 \times 0.05 \times 1 \times 0.2^2}{2} \Rightarrow C_D = 1.07$$



311bha

- 8) A wind of velocity 40 m/s is blowing against an electricity pole 50 cm in diameter and 5 m high. Obtain the drag and the maximum bending moment acting on the pole. Assume that the drag coefficient is 0.6 and the $\rho_{\text{air}} = 1.205 \text{ kg/m}^3$.

Solution;

$$d = 50 \text{ cm} = 0.5 \text{ m}.$$

$$h = 5 \text{ m}.$$

$$C_D = 0.6.$$

$$V = 40 \text{ m/s}; \rho = 1.205 \text{ kg/m}^3$$

$$F_D = C_D \cdot \frac{\rho A V^2}{2}$$

$$= 0.6 \times 1.205 \times (0.5 \times 5) \times 40^2$$

$$= 1446 \text{ N}.$$

$$\text{moment about base} = F_D \times 2.5 = 1446 \times 2.5 = 3615 \text{ Nm}.$$

9. A kite of dimension 0.6×0.6 weighing 5 N flies in equilibrium when it makes an angle of 10° with the horizontal in a wind of speed 40 km/hr and density 1.2 kg/m^3 . If the string attached to it makes an angle of 45° with the horizontal and experiences a tension of 20 N , calculate the lift and drag coefficients.

Solution;

$$\text{Area (A)} = 3.6 \text{ m}^2.$$

$$W = 5 \text{ N}.$$

As the kite is in equilibrium;

$$F_D = 20 \cos 45^\circ.$$

$$\frac{C_D \rho A V^2}{2} = 20 \times \cos 45^\circ.$$

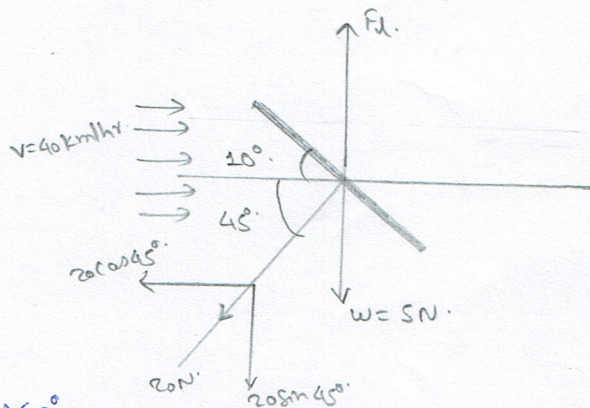
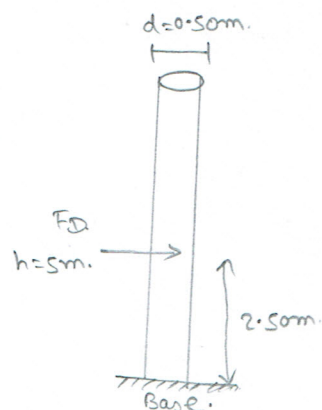
$$\therefore C_D \times 0.6 \times 0.6 \times 1.2 \times 11^2 = 20 \cos 45^\circ$$

$$\therefore C_D = 0.53.$$

$$\text{and, } F_L = W + 20 \sin 45^\circ = 5 + 20 \sin 45^\circ.$$

$$\therefore C_L \times 1.2 \times 0.6^2 \times 11^2 = 5 + \frac{20}{\sqrt{2}}$$

$$\therefore C_L = 0.72.$$



Abhash

Abhash
Acharya.

10. A smooth spherical body of diameter 12cm is travelling at a velocity of 30m/s in windless open area under the condition at 20°C temperature and standard atmospheric pressure. Obtain the drag at the sphere.

Solution;

$$d = 12\text{cm} = 0.12\text{m}.$$

$$V = 30\text{m/s}.$$

$$\mu = 1.8 \times 10^{-5} \text{Pa.s}$$

$$\rho = 1.2\text{kg/m}^3$$

$$Re = \frac{\rho V d}{\mu} = \frac{1.2 \times 30 \times 0.12}{1.8 \times 10^{-5}} = 2.4 \times 10^5.$$

$$\text{For } Re > 10^5, C_D = 0.2.$$

$$\therefore F_D = \frac{C_D \rho A V^2}{2} = 0.2 \times 12 \times \pi \times \frac{0.12^2}{4} \times \frac{30^2}{2} = 1.22\text{N}.$$

11. An oblong barge of length 10m, width 2.5m and draft 0.25m is going up a river at a relative velocity 1.5m/s to the water flow. What are the friction resistance suffered by the barge and power necessary for navigation, assuming a water temperature 20°C.

Solution;

$$l = 10\text{m}, b = 2.5\text{m}.$$

$$\text{draft} = 0.25\text{m}.$$

$$V = 1.5\text{m/s}$$

$$C_D = 1.19.$$

$$\text{Area (A)} = 10 \times 2.5 = 25\text{m}^2.$$

we have;

$$F_D = \frac{\rho A V^2}{2} \times C_D = 1000 \times 25 \times \frac{1.5^2}{2} \times 1.19 = 33468.75\text{N}.$$

$$P_D = F_D \times V = 33468.75 \times 1.5 = 50203.125\text{Watt}.$$

$$T = F_D \times 0.25 = 33468.75 \times 0.25 = 8367.18\text{Nm}.$$

Alibhaskar

12. A 1.5m diameter spherical balloon filled with helium of density one fifth that of air is let go from a meteorological observatory. If the weight of empty balloon is 50N, determine the steady velocity of ascent under normal atmospheric condition taking drag coefficient as 0.8.

Solution;

$$\text{diameter } (d) = 1.5\text{m.}$$

$$\rho_{He} = \frac{1}{5} \rho_{air} = \frac{1}{5} \times 1.2 = 0.24 \text{ kg/m}^3.$$

$$C_D = 0.8.$$

$$\text{weight of empty balloon} = 50\text{N.}$$

$$\text{weight of filled balloon} = 50 + \frac{\pi d^3}{6} \times 0.24 \times 9.81.$$

$$\therefore W = 54.16\text{N.}$$

$$\text{Buoyant force } (F_B) = \frac{\pi d^3}{6} \times \rho_{air} \times g.$$

$$= \frac{1.5^3 \times \pi}{6} \times 1.2 \times 9.81$$

$$= 20.8\text{N.}$$

$$F_D = C_D \frac{\rho A V^2}{2} = 0.8 \times 1.2 \times \pi \times \frac{1.5^2}{4} \times \frac{V^2}{2}$$

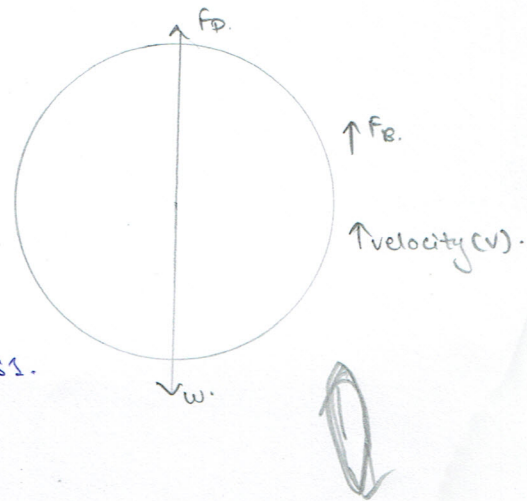
$$= 0.848 V^2$$

we have,

$$W = F_D + F_B.$$

$$\text{or, } 54.16 = 0.848 V^2 + 20.8.$$

$$\therefore V = 6.27 \text{ m/s.}$$



21/5/2021

13. A drag force of 42N is desired at $Re = 10^5$ on a 2m long cylinder in a 20°C atmospheric air flow. What velocity should be selected and what should be the cylinder diameter?

Solution;

For $Re = 10^5$, $C_D = 0.6$.

$$F_D = 42\text{N}.$$

$$L = 2\text{m}.$$

$$\rho = 1.2\text{kg/m}^3$$

$$\mu = 1.8 \times 10^{-5}\text{Pa.s}.$$

Now,

$$Re = \frac{\rho V D}{\mu}$$

$$\text{or, } 10^5 = \frac{1.2 \times V \times D}{1.8 \times 10^{-5}}$$

$$\text{or, } VD = 1.5.$$

$$\text{or, } D = \frac{1.5}{V} \quad \text{--- a)}$$

Also,

$$F_D = \frac{C_D \rho A V^2}{2}$$

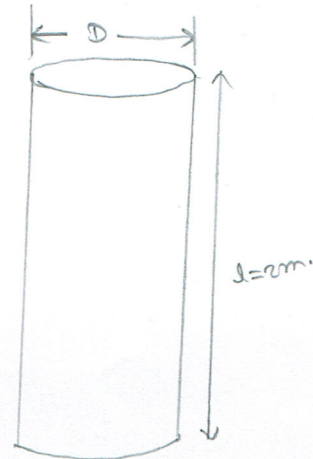
$$\text{or, } 42 = \frac{0.6 \times 1.2 \times (2 \times D) \times V^2}{2}$$

$$\text{or, } DV^2 = 58.33.$$

$$\text{or, } \frac{1.5}{V} \times V^2 = 58.33 \quad [\text{From a)].}$$

$$\therefore V = 38.88\text{ m/s}.$$

$$\therefore D = \frac{1.5}{38.88} = 0.0385\text{ m}.$$



311bhar

16. Experiments were carried out in a wind tunnel with wind speed of 50 km/hr on a flat plate of 2m long and 1m wide. The density of air is 1.25 kg/m^3 and the plate is kept at such an angle that the coefficient of lift and drag are 0.75 and 0.15 respectively. Determine the lift force, drag force, resultant force and power exerted by the air stream on the plate.

Solution;

$$V = 13.89 \text{ m/s.}$$

$$L = 2 \text{ m.}$$

$$b = 1 \text{ m.}$$

$$\rho = 1.25 \text{ kg/m}^3.$$

$$C_L = 0.75.$$

$$C_D = 0.15.$$

$$F_D = \frac{C_D \rho A V^2}{2} = \frac{0.15 \times 2 \times 1 \times 13.89^2}{2} \times 1.25 = 33.28 \text{ N.}$$

$$F_L = \frac{C_L \rho A V^2}{2} = \frac{0.75 \times 1.25 \times 2 \times 1 \times 13.89^2}{2} = 166.4 \text{ N.}$$

$$F = \sqrt{F_D^2 + F_L^2} = \sqrt{33.28^2 + 166.4^2} \\ = 169.7 \text{ N.}$$

$$\theta = \tan^{-1} \frac{F_L}{F_D} = \tan^{-1} \left(\frac{166.4}{33.28} \right) = 78.7^\circ.$$

$$\text{Power} = F_D \times \text{velocity.}$$

$$= 33.28 \times 13.89$$

$$= 462.26 \text{ watt.}$$

Zillbhar

17. Determine the maximum load that a 6m diameter parachute may carry if the maximum permissible velocity is 10 m/s. The coefficient of drag may be assumed as 1.2.

Solution;

At steady condition;

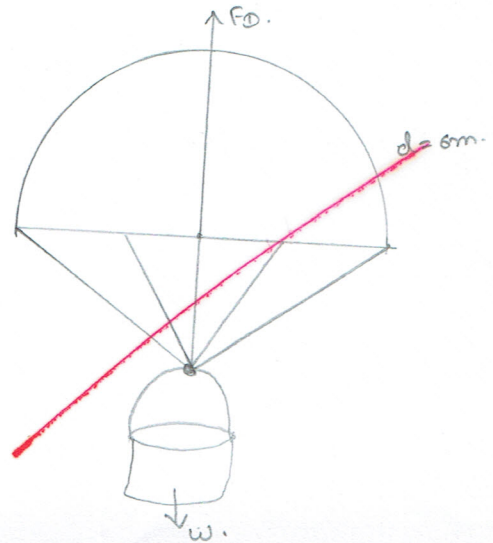
$$W = F_D$$

$$\text{or } W = \frac{C_D \times \rho \times A \times v^2}{2}$$

$$\text{or } W = 1.2 \times 1.2 \times \pi \times \frac{6^2}{4} \times \frac{10^2}{2}$$

$$\text{or } W = 1.44 \times \frac{22}{7} \times 4.5 \times 10^2$$

$$\therefore W = 2035.75 \text{ N}$$



18. A paratrooper and parachute together weigh 1 kN. Estimate its rate of descent if the parachute diameter is 7m and the drag coefficient is assumed to be 1.2. Given that the density of air is 1.22 kg/m³.

Solution;

$$W = 1 \text{ kN} = 1000 \text{ N}$$

$$d = 7 \text{ m}$$

$$C_D = 1.2$$

$$\rho_{\text{air}} = 1.22 \text{ kg/m}^3$$

At steady conditions;

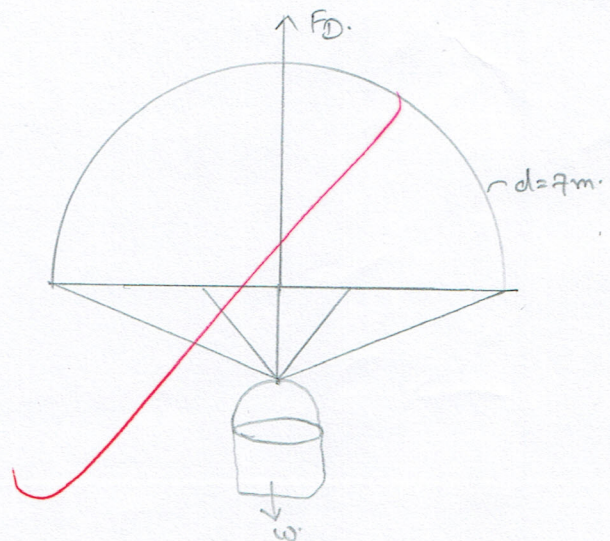
$$W = F_D$$

$$\text{or } W = \frac{C_D \times \rho \times A \times v^2}{2}$$

$$\text{or } W = 1.2 \times 1.22 \times \pi \times \frac{7^2}{4} \times \frac{v^2}{2}$$

$$\text{or } 1000 = 1.44 \times 22 \times \frac{7}{4} \times \frac{v^2}{2}$$

$$\therefore v = 5.9 \text{ m/s}$$



Zilbhar

19. An aeroplane of mass 500kg and wing area 10m^2 is flying in a straight and level flight at a speed of 45m/s near the sea level. Estimate the lift coefficient of the airfoil using.

Solution;

$$V = 45\text{m/s}$$

$$W = 4905\text{N}$$

$$A = 10\text{m}^2$$

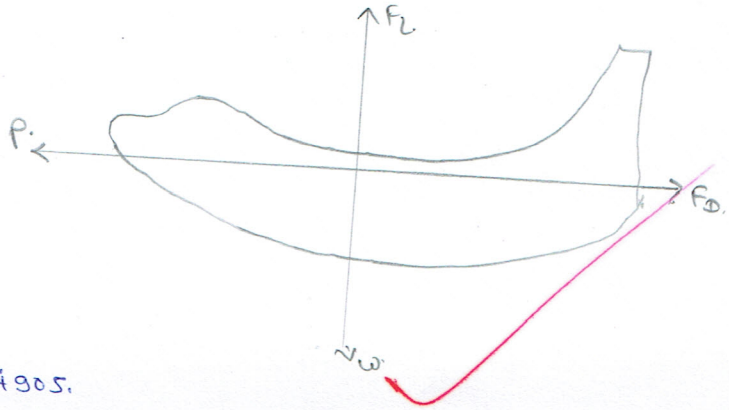
At steady conditions;

$$W = F_L$$

$$\text{or, } W = C_L \times \frac{\rho A V^2}{2}$$

$$\text{or, } C_L \times \frac{1.2 \times 10 \times 45^2}{2} = 4905$$

$$\therefore C_L = 0.4$$



20. A wing of a small aeroplane is rectangular in plan having a span of 10m and a chord of 1.2m . In straight and level flight at 240km/hr the total aerodynamic force acting on the wing is 20kN . If the lift/drag ratio is 10, calculate the coefficient of lift and the total weight the aeroplane can carry. Take air density 1.2kg/m^3 .

Solution;

$$V = 240\text{km/hr}$$

$$= 240 \times \frac{5}{18} \text{m/s}$$

$$= 66.67\text{m/s}$$

$$\rho_{\text{air}} = 1.2\text{kg/m}^3$$

$$\text{Total aerodynamic force on wing} = 20\text{kN} = 20 \times 10^3 \text{N}$$

$$\text{Area (A)} = 1.2 \times 10 = 12\text{m}^2$$

$$\frac{F_L}{F_D} = 10$$

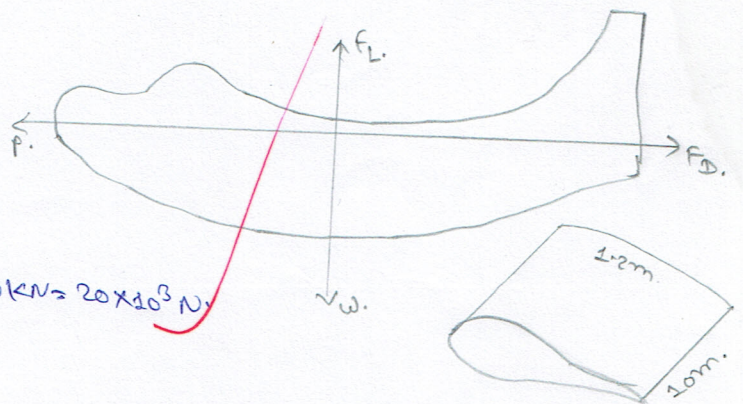
For horizontal flight, aerodynamic force acting on the wing represents the lift force.

$$\therefore F_L = 20\text{kN} \text{ and } F_D = \frac{20}{10} = 2\text{kN}$$

$$F_L = W = 20\text{kN}$$

$$\text{Also, } F_L = C_L \times \frac{\rho A V^2}{2} \Rightarrow 20 \times 10^3 = C_L \times 1.44 \times 10 \times \frac{66.67^2}{2} \Rightarrow C_L = 0.82$$

311bha



21. An aircraft with a mass including payload of 1000kg is designed to cruise at speed of 80 m/s at 10 km. The effective wing area is approximately 15 m². Determine the lift coefficient and the angle of attack. What power is required by the airfoil during cruise? Assume a conventional airfoil.

Solution;

At 10 km;

$$\rho = 0.414 \text{ kg/m}^3$$

$$g = 9.776 \text{ m/s}^2$$

$$V = 80 \text{ m/s}$$

$$h = 10 \text{ km}$$

$$A = 15 \text{ m}^2$$

Now,

$$W = F_L$$

$$\Rightarrow 9776 = \frac{C_L \times 0.414 \times 15 \times 80^2}{2}$$

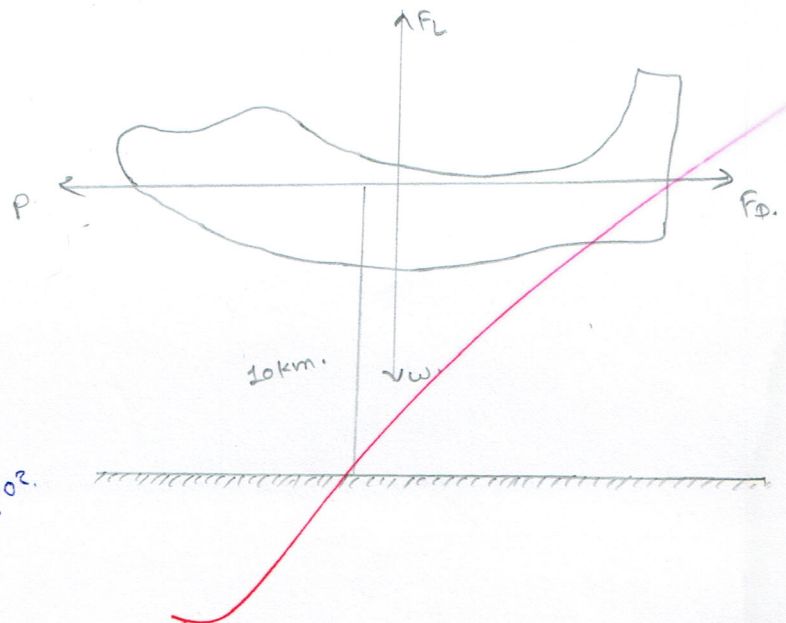
$$\Rightarrow C_L = 0.49 \approx 0.5$$

$$\text{For } C_L = 0.5, C_D = 0.006$$

$$F_D = C_D \times \frac{\rho A V^2}{2} = 0.006 \times 0.414 \times \frac{15 \times 80^2}{2} = 119.232 \text{ N}$$

$$\therefore P = F_D \times V = 119.232 \times 80 = 9538.56 \text{ W}$$

$$\therefore \alpha = 4^\circ$$



311bha

22. A 1500 kg aircraft is designed to carry a payload of 3000 N when cruising at 80 m/s at 10 km. The effective wing area is 20 m². Assuming a conventional air foil, calculate a) the take off speed if an angle of attack of 10° is desired b) the stall speed when landing c) the power required at cruise if 45% of the power needed to move the airfoil.

Solution;

At 10 km.

$$g = 9.776 \text{ m/s}^2.$$

$$\rho = 1.414 \text{ kg/m}^3$$

$$V = 80 \text{ m/s}.$$

$$A = 20 \text{ m}^2.$$

$$\alpha = 10^\circ.$$

$$W = 1500 \times 9.776 \\ = 14664 \text{ N}.$$

$$\text{Total } W = 14664 + 3000 \\ = 17664 \text{ N. (At 10 km).}$$

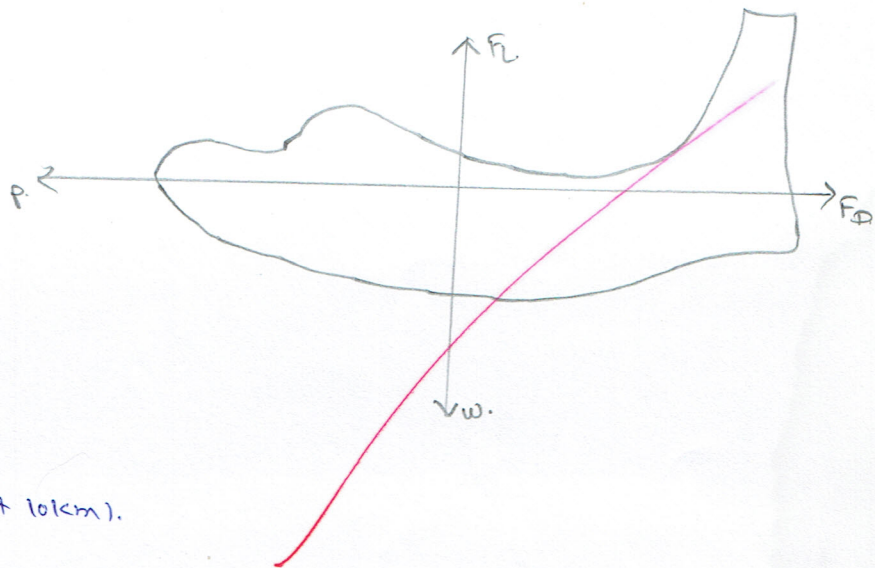
$$\text{At ground; } W = 1500 \times 9.81 + 3000. \\ = 17715 \text{ N}.$$

$$a) F_L = C_L \times \frac{\rho A V^2}{2} = W.$$

$$\text{At } \alpha = 10^\circ; C_L = 1.2.$$

$$\therefore 17715 = 1.2 \times 1.2 \times 20 \times \frac{V^2}{2}.$$

$$\Rightarrow V = 35.07 \text{ m/s}.$$



Zilbhar

24. An additional load of 6000 N is added to the aircraft of problem 19. Estimate the take off speed, if the angle of attack remains at 10° .

Solution;

$$\text{Weight (W)} = 500 \times 9.81 = 4905 \text{ N} + 6000 \\ = 10905 \text{ N.}$$

$$\text{Area (A)} = 20 \text{ m}^2$$

$$\rho = 1.2 \text{ kg/m}^3$$

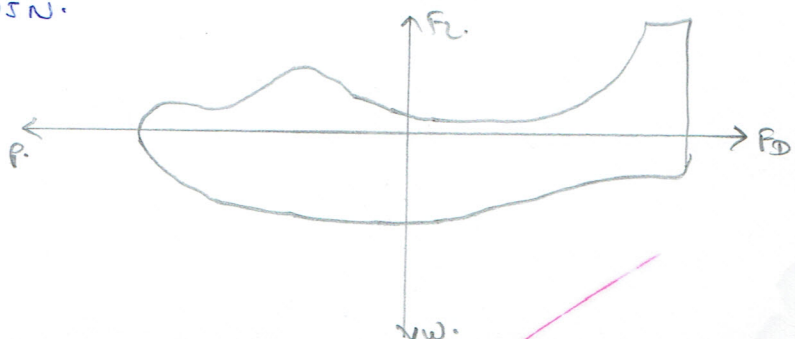
$$\text{At } \alpha = 10^\circ; C_L = 1.2$$

Now;

$$F_L = W = C_L \times \frac{\rho A V^2}{2}$$

$$\text{or } 10905 = 1.2 \times 1.2 \times 20 \times \frac{V^2}{2}$$

$$\therefore V = 38.9 \text{ m/s.}$$



25. A jet plane having a wing area of 20 m^2 and weighing 25 kN flies at 950 km/hr. The engine develops 8500 kW and has a mechanical efficiency of 60%. Determine the lift and drag coefficients for the wind taking the specific weight of air 12 N/m^3 .

Solution;

$$A = 20 \text{ m}^2$$

$$V = 950 \text{ km/hr} = 950 \times \frac{5}{18} \text{ m/s.}$$

$$= 263.89 \text{ m/s.}$$

$$W = 25 \times 10^3 \text{ N.}$$

$$P = 8500 \text{ kW} = 8500 \times 10^3 \text{ W.}$$

$$\eta = 60\% = 0.6.$$

Now;

$$\gamma = 12 \text{ N/m}^3 \Rightarrow \rho = 1.22 \text{ kg/m}^3$$

$$F_L = W \Rightarrow C_L \times \frac{\rho A V^2}{2} = W$$

$$\text{or } C_L \times 1.22 \times 20 \times \frac{263.89^2}{2} = 25 \times 10^3$$

$$\therefore C_L = 0.02944.$$

311bha

Also;

Given that;

$$\text{Gov. of } P = F_D \times V.$$

$$\text{or, } 0.6 \times 8500 \times 10^3 = C_D \times 1.22 \times 20 \times \frac{263.88^2}{2} \times 263.88.$$

$$\therefore C_D = 0.0227.$$

$$\therefore F_D = \frac{C_D \times S A V^2}{2}$$

$$= 0.0227 \times 1.22 \times 20 \times \frac{263.88^2}{2}$$

$$= 19284.06 \text{ N}.$$

26. An aircraft weighing 1000kN when empty has a wing area of 226 m^2 . It is to take off at a velocity of 300 km/hr and a 20° angle of attack. Determine the allowable weight of Cargo and power required for the engine. Take density of air 1.2 kg/m^3 . Assume coefficient of lift for the wing at 20° angle of attack as 1.42 and $C_D = 0.17$.

Solution;

$$A = 226 \text{ m}^2$$

$$V = 300 \text{ km/hr} = 83.33 \text{ m/s}.$$

$$\alpha = 20^\circ.$$

$$C_D = 0.17.$$

$$C_L = 1.42.$$

$$\rho = 1.2 \text{ kg/m}^3.$$

Now;

$$F_L = W = \frac{C_L \times S A V^2}{2}$$

$$\text{or, } (x + 1000) \times 10^3 = 1.42 \times 1.2 \times 226 \times \frac{83.33^2}{2}$$

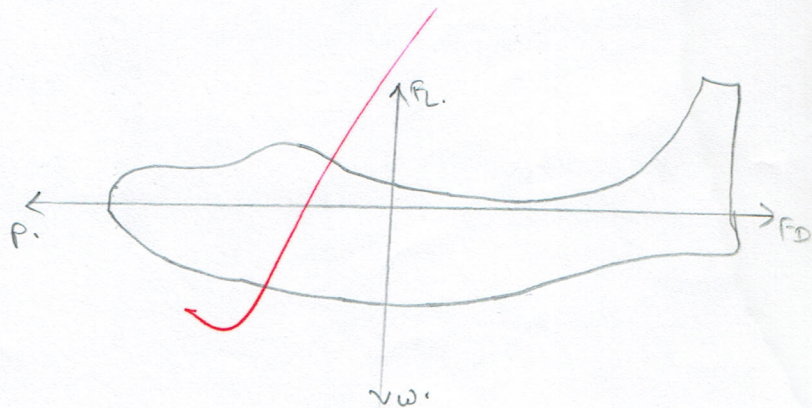
$$\therefore x = 337.06 \text{ kN}.$$

$$\text{Also; } P = F_D \times V.$$

$$= \frac{C_D \times S A V^2}{2} \times V.$$

$$= 0.17 \times 1.2 \times 226 \times \frac{83.33^3}{2}$$

$$= 13338.67 \times 10^3 \text{ watt}.$$



311bha

Tutorial 9

Similitude and Physical Modeling

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1. By Rayleigh method of dimension show that the resistance R to the motion of sphere of diameter D , falling through a viscous fluid of viscosity μ , and density ρ , with velocity v is given by $R = \rho D^2 v^2 \phi \left(\frac{\mu}{\rho v D} \right)$.

Solution;

$$R = f(D, \mu, \rho, v).$$

$$R = K D^a \mu^b \rho^c v^d.$$

L.H.S.

$$[M L T^{-2}]$$

$$M: 1 = b + c.$$

$$T: -2 = -b - d.$$

$$L: 1 = a - b - 3c + d.$$

R.H.S.

$$[L]^a [M L^{-1} T^{-1}]^b [M L^{-3}]^c [L T^{-1}]^d.$$

These are 4 unknowns and 3 equation so expressing 3 unknowns in terms of 4th unknown.

$$c = 1 - b$$

$$d = 2 - b$$

$$a = 1 + b + 3c - d$$

$$= 1 + b + 3 - 3b - 2 + d$$

$$= 2 - b.$$

Putting this value in equation,

$$R = K D^{2-b} \mu^b \rho^{1-b} v^{2-b}.$$

$$= K D^2 v^2 \rho \left(\frac{\mu}{\rho v D} \right)^b.$$

$$R = K \rho D^2 v^2 \phi \left(\frac{\mu}{\rho v D} \right).$$

proved.

Bilbhar

2.

A partially submerged body is towed in water. The resistance R to its motion depends on velocity of body V , density S , viscosity μ , length of body L and acceleration due to gravity g . By Rayleigh's method show that the resistance can be expressed in the form of $R = S L^2 V^2 \phi \left[\frac{\mu}{S V L}, \frac{L g}{V^2} \right]$.

Solution;

$$R = f(S, \mu, L, V, g).$$

$$R = K S^a \mu^b L^c V^d g^e.$$

L.H.S.

$$[M L T^{-2}].$$

$$M: 1 = a + b.$$

$$L: 1 = -3a - b + c + d + e.$$

$$T: -2 = -b - d - 2e.$$

Expressing a, c, d in b and e .

$$a = 1 - b.$$

$$d = 2 - b - 2e.$$

$$1 = -3 + 3b - b + c + 2 - b - 2e + e.$$

$$1 = -1 + b + c - e.$$

$$c = 2 - b + e.$$

Putting these values in equation,

$$R = K S^{1-b} \mu^b L^{2-b+e} V^{2-b-2e} g^e.$$

$$\text{or, } R = K S V^2 L^2 \left[\frac{\mu}{S V L} \right]^b \left[\frac{L g}{V^2} \right]^e.$$

$$\therefore R = K S V^2 L^2 \phi \left[\frac{\mu}{S V L}, \frac{L g}{V^2} \right].$$

proved.

311bha

3. The efficiency η of a fan depends on density ρ , viscosity μ , angular velocity ω , diameter D and the discharge Q . Express efficiency η in terms of dimensionless parameter by Buckingham π -theorem.

$$\eta = \phi\left(\frac{\mu}{D^2 \omega \rho}; \frac{Q}{D^3 \omega}\right).$$

Solution;

$$\eta = f(\rho, \mu, \omega, D, Q).$$

$$\therefore f(\pi_1, \pi_2, \pi_3) = 0.$$

$$n = 6, m = 3.$$

$$n - m = 6 - 3 = 3 = \pi\text{-terms.}$$

$$f(\pi_1, \pi_2, \pi_3).$$

$$\pi_1 = D^{a_1} \omega^{b_1} \rho^{c_1} \eta.$$

$$\pi_2 = D^{a_2} \omega^{b_2} \rho^{c_2} \mu.$$

$$\pi_3 = D^{a_3} \omega^{b_3} \rho^{c_3} Q.$$

π_1

LHS.

$$[M^0 L^0 T^0].$$

RHS.

$$[L]^{a_1} [T^{-1}]^{b_1} [M L^{-3}]^{c_1} [M^0 L^0 T^0].$$

$$0 = c_1; 0 = -b_1; 0 = a_1 - 3c_1 \Rightarrow a_1 = 0.$$

$$\therefore \pi_1 = \eta.$$

π_2

LHS.

$$[M^0 L^0 T^0]$$

RHS.

$$[L]^{a_2} [T^{-1}]^{b_2} [M L^{-3}]^{c_2} [M L^{-1} T^{-1}].$$

$$0 = c_2 + 1 \Rightarrow c_2 = -1; 0 = -b_2 - 1 \Rightarrow b_2 = -1.$$

$$0 = a_2 - 3c_2 - 1 \Rightarrow 0 = a_2 + 3 - 1 \Rightarrow a_2 = -2.$$

$$\therefore \pi_2 = \frac{\mu}{D^2 \omega \rho}.$$

π_3

LHS.

$$[M^0 L^0 T^0].$$

RHS.

$$[L]^{a_3} [T^{-1}]^{b_3} [M L^{-3}]^{c_3} [L^3 T^{-1}].$$

$$0 = c_3; 0 = -b_3 - 1 \Rightarrow b_3 = -1; 0 = a_3 - 3c_3 + 3 \Rightarrow a_3 = -3.$$

$$\therefore \pi_3 = \frac{Q}{D^3 \omega}.$$

$$\therefore f(\pi_1, \pi_2, \pi_3) = 0 \Rightarrow f\left(\eta, \frac{\mu}{D^2 \omega \rho}, \frac{Q}{D^3 \omega}\right) = 0. \quad \therefore \eta = f\left(\frac{\mu}{D^2 \omega \rho}, \frac{Q}{D^3 \omega}\right).$$

3112 21

4. If the velocity v in the fluid flow depends on a dimension L , density ρ , and viscosity μ , show that this implies that the Reynolds number is constant.

Solution;

$$v = f(L, \rho, \mu).$$

$$v = k L^a \rho^b \mu^c; k \text{ is constant.}$$

L.H.S.

$$[L T^{-1}]$$

R.H.S.

$$[L]^a [\rho]^b [\mu]^{-1}]^c.$$

$$0 = b + c.$$

$$\text{or } b + 1 = 0$$

$$\therefore b = -1.$$

$$-1 = -c.$$

$$\therefore c = 1.$$

$$1 = a - 3b - c.$$

$$\Rightarrow 1 = a + 3 - 1$$

$$\Rightarrow a = -1.$$

$$\therefore v = \frac{k \mu}{L \rho} \Rightarrow k = \frac{v L \rho}{\mu} = \text{Re constant.}$$

proved.

5. Assume that the velocity v of fall of an object depends on height H through which it falls, the gravity g , and the mass m of the object. Find the expression for v .

Solution;

$$v = f(H, g, m).$$

$$v = k H^a g^b m^c; k \text{ is constant.}$$

L.H.S.

$$[L T^{-1}]$$

R.H.S.

$$[L]^a [L T^{-2}]^b [m]^c$$

$$0 = c; -1 = -2b$$

$$\Rightarrow b = 1/2.$$

$$1 = a + b$$

$$\therefore a = 1/2.$$

$$v = k H^{1/2} g^{1/2}.$$

$$v = k \sqrt{gH}.$$

$$\therefore v = \sqrt{\frac{gH}{c}}; c \text{ is constant.}$$

Bilbhar

6. Determine an expression for the velocity of liquid issuing from a hole in the side of an open tank if the velocity depends on the height H of the hole from the free surface, the viscosity μ and the density ρ , gravity g , and the hole diameter D , keeping H, ρ and g as repeating variables.

Solution;

$$V = f(H, \mu, \rho, g, D).$$

$$f'(V, H, \mu, \rho, g, D) = 0.$$

$$n=6, m=3; \text{ 7 terms } = n-m = 6-3=3.$$

$$f'(x_1, x_2, x_3) = 0.$$

$$x_1 = H^{a_1} \rho^{b_1} g^{c_1} \mu.$$

$$x_2 = H^{a_2} \rho^{b_2} g^{c_2} D.$$

$$x_3 = H^{a_3} \rho^{b_3} g^{c_3} V.$$

x_1

$$[M^0 L^0 T^0] [L]^{a_1} [M L^{-3}]^{b_1} [L T^{-2}]^{c_1} [M L^{-1} T^{-1}]$$

$$0 = b_1 + 1.$$

$$\Rightarrow b_1 = -1.$$

$$0 = -2c_1 - 1.$$

$$\Rightarrow c_1 = -1/2.$$

$$0 = a_1 - 3b_1 + c_1 - 1.$$

$$\Rightarrow a_1 = -3/2.$$

$$\therefore x_1 = H^{-3/2} \rho^{-1} g^{-1/2} \mu.$$

$$\text{ x_2 } [M^0 L^0 T^0] [L]^{a_2} [M L^{-3}]^{b_2} [L T^{-2}]^{c_2} [L].$$

$$0 = b_2; 0 = -2c_2 \Rightarrow c_2 = 0; 0 = a_2 - 3b_2 - 2c_2 + 1.$$

$$a_2 = -1.$$

$$x_2 = \frac{D}{H}.$$

$$\text{ x_3 } [M^0 L^0 T^0] [L]^{a_3} [M L^{-3}]^{b_3} [L T^{-2}]^{c_3} [L T^{-1}]$$

$$0 = b_3; 0 = -2c_3 - 1 \Rightarrow c_3 = -1/2.$$

$$0 = a_3 - 3b_3 + c_3 + 1$$

$$\Rightarrow 0 = a_3 - 0 - 1/2 + 1$$

$$\therefore a_3 = -1/2. \Rightarrow x_3 = \frac{V}{\sqrt{gH}}.$$

$$f'(H^{-3/2} \rho^{-1} g^{-1/2} \mu; \frac{D}{H}; \frac{V}{\sqrt{gH}}) = 0 \quad \therefore \frac{V}{\sqrt{gH}} = f''\left[\frac{\mu}{\sqrt{gH} H^{3/2}}; \frac{D}{H}\right].$$

31/12/24

7. The flow rate Q in an open channel depends on the hydraulic radius R , the cross-sectional area A , the wall roughness height e , gravity g , and the bed slope s . Relate Q to the other variables (In this problem two basic dimension only so take R and g as repeating variables).

Solution;

$$Q = f(R, A, e, g, s).$$

$$f'(Q, R, A, e, g, s) = 0.$$

$$n=6, m=2 \Rightarrow \text{7 terms} = n-m = 6-2 = 4.$$

$$f'(Q, R, A, e, g, s) = 0.$$

$$\pi_1 = R^{a_1} g^{b_1} Q.$$

$$\pi_2 = R^{a_2} g^{b_2} A.$$

$$\pi_3 = R^{a_3} g^{b_3} e.$$

$$\pi_4 = R^{a_4} g^{b_4} s.$$

$$\pi_1: [m^0 l^0 t^0] [L]^{a_1} [L T^{-2}]^{b_1} [L^3 T^{-1}].$$

$$0=0; \quad 0 = -2b_1 - 1, \quad 0 = a_1 + b_1 + 3$$

$$\Rightarrow b_1 = -1/2, \quad a_1 = -5/2.$$

$$\pi_2: [m^0 l^0 t^0] [L]^{a_2} [L T^{-2}]^{b_2} [L^2].$$

$$0=0; \quad 0 = -2b_2, \quad 0 = a_2 + 2b_2$$

$$\Rightarrow b_2 = 0, \quad a_2 = -2.$$

$$\pi_2 = \frac{A}{R^2}.$$

$$\pi_3: [m^0 l^0 t^0] [L]^{a_3} [L T^{-2}]^{b_3} [L].$$

$$0=0; \quad 0 = -2b_3, \quad 0 = a_3 + 1$$

$$\Rightarrow b_3 = 0, \quad a_3 = -1.$$

$$\pi_3 = \frac{e}{R}.$$

$$\pi_4: [m^0 l^0 t^0] [L]^{a_4} [L T^{-2}]^{b_4} [m^0 l^0 t^0].$$

$$0=0; \quad 0 = -2b_4, \quad 0 = a_4.$$

$$\Rightarrow b_4 = 0, \quad a_4 = 0.$$

$$\therefore f' \left(\frac{Q}{\sqrt{gR^3}}, \frac{A}{R^2}, \frac{e}{R}, s \right) = 0.$$

$$\frac{Q}{\sqrt{gR^3}} = f'' \left(\left(\frac{A}{R^2} \right), \left(\frac{e}{R} \right), s \right).$$

Zillbhan

2). Using Buckingham's π -theorem, show that velocity through a circular orifice is given by $V = \sqrt{2gH} \phi \left[\frac{D}{H}; \frac{\mu}{\sqrt{H}S} \right]$ where, H is the head over orifice, D - diameter of orifice, μ - viscosity, S - density, and g - acceleration due to gravity. Take H, V and S as repeating variables.

Solution;

$$V = f(H, D, \mu, S, g)$$

$$f'(V, H, D, \mu, S, g) = 0$$

$$n=6, m=3, \pi\text{-terms} = n-m = 6-3 = 3.$$

$$\phi'(\pi_1, \pi_2, \pi_3) = 0$$

$$\pi_1 = V^{a_1} H^{b_1} S^{c_1} D$$

$$\pi_2 = V^{a_2} H^{b_2} S^{c_2} \mu$$

$$\pi_3 = V^{a_3} H^{b_3} S^{c_3} g$$

$$\pi_1; [M^0 L^0 T^0] \quad [L T^{-1}]^{a_1} [L]^{b_1} [M L^{-3}]^{c_1} [L]$$

$$0 = c_1; \quad 0 = -a_1 - 3c_1 \Rightarrow a_1 = 0$$

$$0 = a_1 + b_1 - 3(c_1 + 1) \Rightarrow b_1 = -1$$

$$\therefore \pi_1 = \frac{D}{H}$$

$$\pi_2; [M^0 L^0 T^0] \quad [L T^{-1}]^{a_2} [L]^{b_2} [M L^{-3}]^{c_2} [M L^{-1} T^{-1}]$$

$$0 = 1 + c_2 \Rightarrow c_2 = -1; \quad 0 = -a_2 - 1 \Rightarrow a_2 = -1$$

$$0 = a_2 + b_2 - 3(c_2 - 1) \Rightarrow b_2 = -1$$

$$\therefore \pi_2 = \frac{\mu}{\sqrt{H}S}$$

$$\pi_3; [M^0 L^0 T^0] \quad [L T^{-1}]^{a_3} [L]^{b_3} [M L^{-3}]^{c_3} [L T^{-2}]$$

$$0 = c_3; \quad 0 = -a_3 - 2 \Rightarrow a_3 = -2$$

$$0 = a_3 + b_3 - 3(c_3 + 1) \Rightarrow 0 = -2 + b_3 - 0 + 1$$

$$\therefore b_3 = 2$$

$$\therefore \pi_3 = \frac{Hg}{V^2}$$

$$\phi' \left(\frac{D}{H}, \frac{\mu}{\sqrt{H}S}, \frac{Hg}{V^2} \right) = 0$$

$$\therefore V = \sqrt{2gH} \phi \left[\frac{D}{H}; \frac{\mu}{\sqrt{H}S} \right] \text{ constant } K = \sqrt{2}$$

3/1/2021

9. The pressure difference ΔP in a pipe of diameter D , length L , depends on velocity of flow V , viscosity of fluid μ , density of fluid ρ , and roughness of pipe K . Using Buckingham's π -theorem, obtain an expression for ΔP . Take dimension for roughness in length. Take D , V and ρ as repeating variables.

Solution:

$$\Delta P = f(D, L, V, \mu, \rho, K)$$

$$f'(\Delta P, D, L, V, \mu, \rho, K) = 0$$

$$n = 7, m = 3$$

$$\pi \text{ terms} = 7 - 3 = 4$$

$$f'(\pi_1, \pi_2, \pi_3, \pi_4) = 0$$

$$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} \Delta P$$

$$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} L$$

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} \mu$$

$$\pi_4 = D^{a_4} V^{b_4} \rho^{c_4} K$$

π_1 : $[M^0 L^0 T^0] [L]^{a_1} [L T^{-1}]^{b_1} [M L^{-3}]^{c_1} [M L^{-1} T^{-2}]$

$$0 = a_1 + 1 \quad ; \quad 0 = -b_1 - 2 \quad ; \quad 0 = a_1 + b_1 - 3c_1 - 1$$

$$\Rightarrow a_1 = -1 \quad \Rightarrow b_1 = -2 \quad \Rightarrow 0 = a_1 - 2 + 3 - 1$$

$$\Rightarrow a_1 = 0$$

$$\therefore \pi_1 = \frac{\Delta P}{V^2 \rho}$$

π_2 : $[M^0 L^0 T^0] [L]^{a_2} [L T^{-1}]^{b_2} [M L^{-3}]^{c_2} [L]$

$$0 = c_2 \quad ; \quad 0 = -b_2 \quad ; \quad 0 = a_2 + b_2 - 3c_2 + 1 \Rightarrow a_2 = -1$$

$$\pi_2 = \frac{L}{D}$$

π_3 : $[M^0 L^0 T^0] [L]^{a_3} [L T^{-1}]^{b_3} [M L^{-3}]^{c_3} [M L^{-1} T^{-1}]$

$$0 = c_3 + 1 \Rightarrow c_3 = -1 \quad ; \quad 0 = -b_3 - 1 \Rightarrow b_3 = -1$$

$$0 = a_3 + b_3 - 3c_3 - 1 \Rightarrow 0 = a_3 - 1 + 3 - 1 \Rightarrow a_3 = -1$$

$$\pi_3 = \frac{\mu}{D V \rho}$$

π_4 : $[M^0 L^0 T^0] [L]^{a_4} [L T^{-1}]^{b_4} [M L^{-3}]^{c_4} [L]$

$$0 = c_4 \quad ; \quad 0 = -b_4 \quad ; \quad 0 = a_4 + b_4 - 3c_4 + 1 \Rightarrow a_4 = -1$$

$$\therefore \pi_4 = \frac{K}{D}$$

$$\therefore \Delta P = V^2 \rho \phi \left[\frac{L}{D} ; \frac{\mu}{D V \rho} ; \frac{K}{D} \right]$$

311bha

10. The resistance force F of a supersonic plane during flight can be considered as dependent upon the length of the aircraft L , velocity V , air viscosity μ , air density ρ and bulk modulus of air K . Express the functional relationship between these variables and the resisting force. Take L, V and ρ as repeating variables.

Solution;

$$F = f(L, V, \mu, \rho, K).$$

$$f^1[F, L, V, \mu, \rho, K] = 0.$$

$$n = 6; m = 3 \Rightarrow 3 \text{ terms} = n - m = 3.$$

$$f^1(\pi_1, \pi_2, \pi_3) = 0.$$

$$\pi_1 = L^{a_1} V^{b_1} \rho^{c_1} F.$$

$$\pi_2 = L^{a_2} V^{b_2} \rho^{c_2} \mu.$$

$$\pi_3 = L^{a_3} V^{b_3} \rho^{c_3} K.$$

$$\pi_1; [M^0 L^0 T^0] [L]^{a_1} [L T^{-1}]^{b_1} [M L^{-3}]^{c_1} [M L T^{-2}].$$

$$0 = c_1 + 1 \Rightarrow c_1 = -1; 0 = -b_1 - 2 \Rightarrow b_1 = -2.$$

$$0 = a_1 + b_1 - 3c_1 + 1 \Rightarrow 0 = a_1 - 2 + 3 + 1 \Rightarrow a_1 = -2.$$

$$\pi_1 = \frac{F}{L^2 V^2 \rho}.$$

$$\pi_2; [M^0 L^0 T^0] [L]^{a_2} [L T^{-1}]^{b_2} [M L^{-3}]^{c_2} [M L^{-1} T^{-1}].$$

$$0 = c_2 + 1 \Rightarrow c_2 = -1; 0 = -b_2 - 1 \Rightarrow b_2 = -1.$$

$$0 = a_2 + b_2 - 3c_2 - 1 \Rightarrow 0 = a_2 - 1 + 3 - 1 \Rightarrow a_2 = -1.$$

$$\therefore \pi_2 = \frac{\mu}{L V \rho}.$$

$$\pi_3; [M^0 L^0 T^0] [L]^{a_3} [L T^{-1}]^{b_3} [M L^{-3}]^{c_3} [M L^{-1} T^{-2}].$$

$$0 = c_3 + 1 \Rightarrow c_3 = -1; 0 = -b_3 - 2 \Rightarrow b_3 = -2.$$

$$0 = a_3 + b_3 - 3c_3 - 1 \Rightarrow 0 = a_3 - 2 + 3 + (-1) \Rightarrow a_3 = 0.$$

$$\therefore \pi_3 = \frac{K}{\sqrt{2} \rho}.$$

$$f^1 \left[\frac{F}{L^2 V^2 \rho}, \frac{\mu}{L V \rho}, \frac{K}{\sqrt{2} \rho} \right] = 0.$$

$$\therefore F = \rho L^2 V^2 \phi \left[\frac{\mu}{L V \rho}, \frac{K}{\sqrt{2} \rho} \right].$$

Zillbhan

11. Derive on the basis of dimensional analysis suitable parameters to present the thrust developed by a propeller. Assume that the thrust T depends upon the angular velocity ω , speed of advance V , diameter D , viscosity μ , density ρ , and elasticity of the fluid medium which can be represented by the speed of sound C in the medium, take D, V and ρ as repeating variables.

Solution;

$$F = f(\omega, V, D, \mu, \rho, C).$$

$$f_1(F, \omega, V, D, \mu, \rho, C) = 0.$$

$$n=7, m=3 \Rightarrow \text{terms} = 7-3=4.$$

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4) = 0.$$

$$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} F.$$

$$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} \omega.$$

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} \mu.$$

$$\pi_4 = D^{a_4} V^{b_4} \rho^{c_4} C.$$

$$\underline{\pi_1}; [M^0 L^0 T^0] [L]^a [L T^{-1}]^b [M L^{-3}]^c [M L T^{-2}].$$

$$0 = a + 1 \Rightarrow a = -1; 0 = -b - 2 \Rightarrow b = -2.$$

$$0 = a + b - 3c + 1 \Rightarrow 0 = a - 2 + 3 + 1 \Rightarrow a = -2.$$

$$\pi_1 = \frac{F}{D^2 V^2 \rho}.$$

$$\underline{\pi_2}; [M^0 L^0 T^0] = [L]^a [L T^{-1}]^b [M L^{-3}]^c [T^{-1}].$$

$$0 = c; 0 = -b - 1 \Rightarrow b = -1; 0 = a + b - 3c \Rightarrow 0 = a - 1 + 0 \Rightarrow a = 1.$$

$$\therefore \pi_2 = \frac{\omega D}{V}.$$

$$\underline{\pi_3}; [M^0 L^0 T^0] [L]^a [L T^{-1}]^b [M L^{-3}]^c [M L^{-1} T^{-1}].$$

$$0 = c + 1 \Rightarrow c = -1; 0 = -b - 1 \Rightarrow b = -1.$$

$$0 = a + b - 3c - 1 \Rightarrow 0 = a - 1 + 3 - 1 \Rightarrow a = -1.$$

$$\therefore \pi_3 = \frac{\mu}{D \rho V} = \frac{1}{Re}.$$

$$\underline{\pi_4}; [M^0 L^0 T^0] = [L]^a [L T^{-1}]^b [M L^{-3}]^c [L T^{-1}].$$

$$0 = c; 0 = -b - 1 \Rightarrow b = -1; 0 = a + b - 3c + 1 \Rightarrow a = 0.$$

$$\therefore \pi_4 = \frac{C}{V}. \quad \therefore F = \rho V^2 D^2 \phi \left[\frac{1}{Re}, \frac{\omega D}{V}, \frac{C}{V} \right].$$

311bha

12. The drag force F on a golf ball depends on the velocity v , viscosity μ , density ρ , diameter d , dimple depth e , dimple radius r , and concentration C of the dimples measured by the number of dimples per unit area. What expression relates drag force F to the other variables? Take v, ρ and d as repeating variables.

Solution;

$$F = f(v, \mu, \rho, d, e, r, C)$$

$$f(F, v, \mu, \rho, d, e, r, C) = 0$$

$$n = 8; m = 3; \text{ terms } = 8 - 3 = 5.$$

$$f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0$$

$$\pi_1 = v^{a_1} \rho^{b_1} d^{c_1} F$$

$$\pi_2 = v^{a_2} \rho^{b_2} d^{c_2} \mu$$

$$\pi_3 = v^{a_3} \rho^{b_3} d^{c_3} e$$

$$\pi_4 = v^{a_4} \rho^{b_4} d^{c_4} r$$

$$\pi_5 = v^{a_5} \rho^{b_5} d^{c_5} C$$

$$\pi_1: [M^0 L^0 T^0] [L T^{-1}]^{a_1} [M L^{-3}]^{b_1} [L]^c [M L T^{-2}]$$

$$0 = b_1 + 1 \Rightarrow b_1 = -1; 0 = -a_1 - 2 \Rightarrow a_1 = -2; 0 = a_1 - 3b_1 + c_1 + 1 \Rightarrow 0 = -2 + 3 + c_1 + 1 \Rightarrow c_1 = 0$$

$$\therefore \pi_1 = \frac{F}{v^2 \rho d^2}$$

$$\pi_2: [M^0 L^0 T^0] [L T^{-1}]^{a_2} [M L^{-3}]^{b_2} [L]^c [M L^{-1} T^{-1}]$$

$$0 = b_2 + 1 \Rightarrow b_2 = -1; 0 = -a_2 - 1 \Rightarrow a_2 = -1$$

$$0 = a_2 - 3b_2 + c_2 - 1 \Rightarrow 0 = -1 + 3 + c_2 - 1 \Rightarrow c_2 = -1$$

$$\therefore \pi_2 = \frac{\mu}{v \rho d}$$

$$\pi_3: [M^0 L^0 T^0] [L T^{-1}]^{a_3} [M L^{-3}]^{b_3} [L]^c [L]$$

$$0 = b_3; 0 = -a_3; 0 = a_3 - 3b_3 + c_3 + 1 \Rightarrow 0 = 0 - 0 + c_3 + 1 \Rightarrow c_3 = -1$$

$$\therefore \pi_3 = \frac{e}{d}$$

$$\pi_4: [M^0 L^0 T^0] [L T^{-1}]^{a_4} [M L^{-3}]^{b_4} [L]^c [L]$$

$$0 = b_4; 0 = -a_4; 0 = a_4 - 3b_4 + c_4 + 1 \Rightarrow 0 = 0 - 0 + c_4 + 1 \Rightarrow c_4 = -1$$

$$\therefore \pi_4 = \frac{r}{d}$$

2117thad

$$\pi_5 [M^0 L^0 T^0] [L T^{-1}]^{a_5} [M L^{-3}]^{b_5} [L]^c [L^2]$$

$$0 = b_5; 0 = -a_5; 0 = a_5 - 3b_5 + c - 2 \Rightarrow c = 2.$$

$$\therefore \pi_5 = c d^2.$$

$$\therefore f' \left[\frac{F}{\sqrt{s} d^2}, \frac{u}{\sqrt{s} d}, \frac{e}{d}, \frac{\alpha}{d}, c d^2 \right].$$

$$\therefore \frac{F}{\sqrt{s} d^2} = f'' \left(\frac{u}{\sqrt{s} d}, \frac{e}{d}, \frac{\alpha}{d}, c d^2 \right).$$

13.

The lift force F_L on an aircraft depends on the aircraft speed v , the speed of sound c , the fluid density ρ , the airfoil chord length L_c , the airfoil thickness t , and the angle of attack α . Find the expression for the lift force F_L , taking v, ρ, L_c as repeating variables.

Solution;

$$F_L = f(v, \rho, L_c, t, \alpha).$$

$$f'(F_L, v, \rho, L_c, t, \alpha) = 0.$$

$$n = 7, m = 3 \Rightarrow n - m = 7 - 3 = 4.$$

$$\pi_1 = v^{a_1} \rho^{b_1} L_c^{c_1} F_L.$$

$$\pi_2 = v^{a_2} \rho^{b_2} L_c^{c_2} t.$$

$$\pi_3 = v^{a_3} \rho^{b_3} L_c^{c_3} \alpha.$$

$$\pi_4 = v^{a_4} \rho^{b_4} L_c^{c_4} \alpha.$$

$$\pi_1 [M^0 L^0 T^0] [L T^{-1}]^{a_1} [M L^{-3}]^{b_1} [L]^c [M L T^{-2}].$$

$$0 = b_1 + 1 \Rightarrow b_1 = -1; 0 = -a_1 - 2 \Rightarrow a_1 = -2;$$

$$0 = a_1 - 3b_1 + c_1 + 1 \Rightarrow 0 = -2 + 3 + c_1 + 1 \Rightarrow c_1 = -2.$$

$$\pi_1 = \frac{F_L}{\sqrt{s} L_c^2}.$$

$$\pi_2 [M^0 L^0 T^0] [L T^{-1}]^{a_2} [M L^{-3}]^{b_2} [L]^c [L T^{-1}].$$

$$0 = b_2; 0 = -a_2 - 1 \Rightarrow a_2 = -1; 0 = a_2 - 3b_2 + c_2 + 1 \Rightarrow c_2 = 0.$$

$$\therefore \pi_2 = \frac{t}{L_c}.$$

$$\pi_3 [M^0 L^0 T^0] [L T^{-1}]^{a_3} [M L^{-3}]^{b_3} [L]^c [L].$$

$$0 = b_3; 0 = -a_3; 0 = a_3 - 3b_3 + c_3 + 1 \Rightarrow c_3 = -1.$$

$$\therefore \pi_3 = \frac{t}{L_c}.$$

Bibha

$$\pi_4: [m^0 L^0 T^0] [L^{-1}]^{a_4} [m L^{-3}]^{b_4} [L]^{c_4} [m^0 L^0 T^0].$$

$$0 = b_4; 0 = -a_4; 0 = a_4 - 3b_4 + c_4 \Rightarrow c_4 = 0.$$

$$\therefore \pi_4 = \alpha.$$

$$0 = f'(\pi_1, \pi_2, \pi_3, \pi_4)$$

$$\Rightarrow f' \left[\frac{F_L}{s v^2 L^2}, \frac{c}{v}, \frac{t}{L}, \alpha \right] = 0.$$

$$\therefore \frac{F_L}{s v^2 L^2} = f_1 \left[\frac{c}{v}, \frac{t}{L}, \alpha \right].$$

311bharvi

14. Derive an expression for the maximum torque T necessary to rotate an agitator if it depends on the frequency of oscillation f , the angular velocity ω with which the agitator rotates during an oscillation, the diameter of the agitator d , the nominal height of the paddles H , the length of the paddles L , the number of paddles N , the depth of liquid h and density of liquid s , keeping ω and s as repeating variables.

Solution;

$$T = f(f, \omega, d, H, L, N, h, s)$$

$$f''(T, f, \omega, d, H, L, N, h, s) = 0.$$

$$n=9, m=3 \Rightarrow \text{terms} = 6.$$

$$\phi(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6) = 0$$

$$\pi_1 = \omega^{a_1} d^{b_1} s^{c_1} T$$

$$\pi_2 = \omega^{a_2} d^{b_2} s^{c_2} f$$

$$\pi_3 = \omega^{a_3} d^{b_3} s^{c_3} H$$

$$\pi_4 = \omega^{a_4} d^{b_4} s^{c_4} L$$

$$\pi_5 = \omega^{a_5} d^{b_5} s^{c_5} N$$

$$\pi_6 = \omega^{a_6} d^{b_6} s^{c_6} h$$

$$\pi_1: [m^0 L^0 T^0] = [T^{-1}]^{a_1} [L]^{b_1} [m L^{-3}]^{c_1} [m L^2 T^{-2}].$$

$$0 = c_1 + 1 \Rightarrow c_1 = -1; 0 = -a_1 - 2 \Rightarrow a_1 = -2; 0 = b_1 - 3c_1 + 2 \Rightarrow b_1 = -5$$

$$\therefore \pi_1 = \frac{T}{s \omega^2 d^5}.$$

$$\pi_2: [m^0 L^0 T^0] [T^{-1}]^{a_2} [L]^{b_2} [m L^{-3}]^{c_2} [T^{-1}].$$

$$0 = c_2 + 0 \Rightarrow c_2 = 0; 0 = -a_2 - 1 \Rightarrow a_2 = -1; 0 = b_2 - 3c_2 + 0 \Rightarrow b_2 = 0.$$

$$\therefore \pi_2 = f/\omega.$$

$$\pi_3: [M^0 L^0 T^0] [T^{-1}]^{a_3} [L]^{b_3} [M L^{-3}]^{c_3} [L].$$

$$0 = c_3; 0 = -a_3 - 3c_3 \Rightarrow a_3 = 0; 0 = b_3 - 3c_3 + 1 \therefore b_3 = -1.$$

$$\therefore \pi_3 = \frac{H}{d}.$$

$$\pi_4: [M^0 L^0 T^0] [T^{-1}]^{a_4} [L]^{b_4} [M L^{-3}]^{c_4} [L]$$

$$0 = c_4; 0 = -a_4; b_4 = -1 \Rightarrow \pi_4 = \frac{L}{d}.$$

$$\pi_5: [M^0 L^0 T^0] [T^{-1}]^{a_5} [L]^{b_5} [M L^{-3}]^{c_5} [M^0 L^0 T^0].$$

$$0 = c_5; 0 = -a_5 - 3c_5 \Rightarrow a_5 = 0; 0 = b_5 - 3c_5 \times 0 \Rightarrow b_5 = 0.$$

$$\therefore \pi_5 = N.$$

$$\pi_6: [M^0 L^0 T^0] [T^{-1}]^{a_6} [L]^{b_6} [M L^{-3}]^{c_6} [L].$$

$$\therefore \pi_6 = \frac{h}{d}.$$

$$\therefore \Phi \left(\frac{T}{\rho \omega^2 d^5}, \frac{f}{\omega}, \frac{H}{d}, \frac{L}{d}, N, \frac{h}{d} \right)$$

$$\therefore \frac{T}{\rho \omega^2 d^5} = f_1 \left(\frac{f}{\omega}, \frac{H}{d}, \frac{L}{d}, \frac{h}{d}, N \right).$$

15. The flow rate of water over a weir depends on the head of water H , the width of weir b , gravity g , viscosity μ , density ρ , surface tension σ . Relate the flow rate to the other variables (keeping H, g, ρ as repeating variables).

Solution;

$$Q = f(H, b, g, \mu, \rho, \sigma).$$

$$f'(Q, H, b, g, \mu, \rho, \sigma) = 0.$$

$$n=7, m=3 \Rightarrow \pi\text{-terms} = 4.$$

$$f''(\pi_1, \pi_2, \pi_3, \pi_4) = 0.$$

$$\pi_1 = H^{a_1} g^{b_1} \rho^{c_1} Q.$$

$$\pi_2 = H^{a_2} g^{b_2} \rho^{c_2} b.$$

$$\pi_3 = H^{a_3} g^{b_3} \rho^{c_3} \mu.$$

$$\pi_4 = H^{a_4} g^{b_4} \rho^{c_4} \sigma.$$

311bha

$$\underline{r_1} = [m^0 l^0 t^0] [L]^{a_1} [L T^{-2}]^{b_1} [m L^3]^{c_1} [L^3 T^{-1}]$$

$$0 = c_1; 0 = -2b_1(-1) \Rightarrow b_1 = -1/2; 0 = a_1 + b_1 - 3c_1 + 3, \\ \Rightarrow a_1 = -5/2.$$

$$\therefore r_1 = \frac{Q_1}{\sqrt{g H^5}}.$$

$$\underline{r_2} = [m^0 l^0 t^0] [L]^{a_2} [L T^{-2}]^{b_2} [m L^3]^{c_2} [L].$$

$$0 = c_2; 0 = -2b_2 \Rightarrow b_2 = 0; 0 = a_2 + b_2 - 3c_2 + 1 \\ \Rightarrow a_2 = -1.$$

$$\therefore r_2 = \frac{b}{H}.$$

$$\underline{r_3} = [m^0 l^0 t^0] [L]^{a_3} [L T^{-2}]^{b_3} [m L^3]^{c_3} [m L^3 T^{-1}].$$

$$0 = c_3 + 1 \Rightarrow c_3 = -1; 0 = -2b_3 - 1 \Rightarrow b_3 = -1/2.$$

$$0 = a_3 + b_3 - 3c_3 - 1 \Rightarrow 0 = a_3 - 1/2 + 3 - 1 \Rightarrow a_3 = -3/2.$$

$$\therefore r_3 = H^{-3/2} \cdot g^{-1/2} \cdot s^{-1} \mu.$$

$$= \frac{\mu}{8\sqrt{g H^3}}.$$

$$\underline{r_4} = [m^0 l^0 t^0] [L]^{a_4} [L T^{-2}]^{b_4} [m L^3]^{c_4} [m L^3 T^{-2}].$$

$$0 = c_4 + 1 \Rightarrow c_4 = -1; 0 = -2b_4 - 2 \Rightarrow b_4 = -1.$$

$$0 = a_4 + b_4 - 3c_4 - 0 \Rightarrow a_4 - 1 + 3 - 0 = 0$$

$$\Rightarrow a_4 = -2.$$

$$\therefore r_4 = \frac{\sigma}{8g H^2}.$$

$$\therefore f' \left(\frac{Q_1}{\sqrt{g H^5}} + \frac{b}{H} + \frac{\mu}{8\sqrt{g H^3}} + \frac{\sigma}{8g H^2} \right) = 0.$$

$$\therefore Q = \sqrt{g H^5} \phi \left[\frac{b}{H}, \frac{\mu}{8\sqrt{g H^3}}, \frac{\sigma}{8g H^2} \right]$$

311bhaan

16. A force on a component of a 1:10 scale model of a large pump is measured to be 45 N. What force is expected on the prototype component if water is used for both model and prototype: a) with the water at the same temperature; b) with the prototype water at 12°C and the model water at 25°C?

Solution;

Given;
 $L_r = 10$

Model	Prototype
$F = 45 \text{ N}$	$F = ?$
Water	Water

a) at same temperature

$$(Re)_p = (Re)_m$$

$$\left(\frac{\rho V D}{\mu} \right)_p = \left(\frac{\rho V D}{\mu} \right)_m$$

$$\text{or, } \frac{V_p}{V_m} = \frac{D_m}{D_p} = \frac{1}{10}$$

$$\text{or, } \frac{F_p}{F_m} = \frac{(\rho V^2 L^2)_p}{(\rho V^2 L^2)_m} = \left(\frac{V_p}{V_m} \right)^2 \times \left(\frac{L_p}{L_m} \right)^2 = \left(\frac{1}{10} \right)^2 \times 10^2 = 1$$

$$\therefore F_p = F_m = 45 \text{ N}$$

311Bhanu

17. A 1:10 Scale model study of a torpedo is proposed. Prototype speeds of 90 km/hr are to be studied. Should a wind tunnel or a water facility be used?

Solution;

Given; $L_r = 10$

Model

$V_m = ?$

water.

Prototype

$$V_p = 90 \text{ km/hr} = 90 \times \frac{5}{18} \text{ m/s} = 25 \text{ m/s}$$

water.

Torpedo travels under water, viscous force predominates. So, Reynold's number is used.

$$(Re)_p = (Re)_m$$

$$\text{or, } \left(\frac{VL\rho}{\mu} \right)_p = \left(\frac{VL\rho}{\mu} \right)_m$$

$$\text{or, } L_p = L_m; \rho_p = \rho_m$$

$$V_p L_p = V_m L_m$$

$$\Rightarrow V_m = V_p \times \frac{L_p}{L_m}$$

$$= 90 \times 10 = 900 \text{ km/hr.}$$

311bhan

18. The flow over a weir is $2 \text{ m}^3/\text{s}$. A 1:10 model of weir is tested in water channel. a) what flow rate should be used? b) if a force of 12 N is measured on the model, what force would be expected on the prototype?

Solution;

$$L_r = 1:10.$$

Model.

$$Q_m = ?$$

$$F_m = 12 \text{ N}.$$

Prototype.

$$Q_p = 2 \text{ m}^3/\text{sec}.$$

$$F_p = ?$$

In case of weir, Froude no. is used.

$$(Fr)_p = (Fr)_m.$$

$$\text{or, } \left(\frac{V}{\sqrt{g L}} \right)_p = \left(\frac{V}{\sqrt{g L}} \right)_m.$$

$$\text{or, } \frac{V_p^2}{L_p} = \frac{V_m^2}{L_m} \Rightarrow \frac{V_p}{V_m} = L_r^{1/2}.$$

$$\text{a) } \frac{Q_p}{Q_m} = \frac{A_p}{A_m} \times \frac{V_p}{V_m}$$

$$= \left(\frac{L_p}{L_m} \right)^2 \times \frac{V_p}{V_m}$$

$$= \left(\frac{L_p}{L_m} \right)^{5/2}$$

$$= L_r^{5/2}.$$

$$\therefore Q_p = Q_m \times L_r^{2.5}$$

$$\Rightarrow Q_m = \frac{2}{10^{2.5}} = 0.00632 \text{ m}^3/\text{sec}$$

$$\text{b) } \frac{F_p}{F_m} = \frac{m_p g}{m_m g} = \frac{V \times 3}{V \times 8} = \left(\frac{L_p}{L_m} \right)^3 = 10^3.$$

$$\therefore F_p = 12 \times 10^3 = 12000 \text{ N}.$$

311bhard

19. A 1:20 scale model of a propeller on a ship is to be tested in a water channel. what should be the rotational speed of the model if the rotational speed of the propeller is 2000 rpm and; a) the Froude number governs the study? b) the Reynold number governs the study?

Solution;

$$L = 20.$$

model

$$N = ?$$

prototype.

$$N = 2000 \text{ rpm.}$$

$$= \frac{2000}{60} \text{ rps.}$$

a) Froude number's application;

$$(Fr)_p = (Fr)_m.$$

$$\left(\frac{V}{\sqrt{gL}} \right)_p = \left(\frac{V}{\sqrt{gL}} \right)_m.$$

$$\alpha, \frac{V_p^2}{V_m^2} = \frac{L_p}{L_m} = 20.$$

$$\alpha, \frac{N_p^2 \times L_p^2}{N_m^2 \times L_m^2} = 20. \quad [\because V = \omega r = 2\pi N r].$$

$$\alpha, \frac{N_p^2}{N_m^2} = 20 \times \left(\frac{1}{20} \right)^2.$$

$$\Rightarrow N_m = \sqrt{2000^2 \times 10} \\ = 6324.57 \text{ rpm.}$$

b) Reynolds number;

$$(Re)_p = (Re)_m.$$

$$\alpha, \left(\frac{VL}{\mu} \right)_p = \left(\frac{VL}{\mu} \right)_m.$$

$$\alpha, V_p L_p = V_m L_m.$$

$$\alpha, V_m = \frac{V_p L_p}{L_m}.$$

$$\Rightarrow N_m \times L_m = N_p \times L_p \times \frac{L_p}{L_m}$$

$$N_m = 2000 \times 20^2 \text{ rpm.}$$

$$= 200000 \text{ rpm.}$$

311bhan

20. A test is to be performed on a proposed design for a large pump that is to deliver $1.5 \text{ m}^3/\text{sec}$ of water from a 40 cm diameter impeller with a pressure rise of 400 kPa , a model with an 8 cm diameter impeller is to be used. What flow rate should be used and what pressure rise is to be expected? The model fluid is water at the same temperature as the water in the prototype.

Solution;

model
 $Q_m = ?$
 $D_m = 0.08 \text{ m}$
 $(\Delta P)_m = ?$

prototype
 $Q_p = 1.5 \text{ m}^3/\text{s}$
 $D_p = 40 \text{ cm} = 0.4 \text{ m}$
 $(\Delta P)_p = 400 \text{ kPa}$

Now;

$$(f_e)_p = (f_e)_m$$

$$\text{or, } \left(\frac{8VD}{\mu} \right)_p = \left(\frac{8VD}{\mu} \right)_m$$

$$\text{or, } \frac{V_m}{V_p} = \frac{D_p}{D_m} = 5 = L_r^{1/2}$$

$$\therefore L_r = 25$$

$$\frac{Q_m}{Q_p} = \frac{A_m}{A_p} \times \frac{V_m}{V_p} = \frac{D_m^2}{D_p^2} \times \frac{V_m}{V_p}$$

$$= \frac{0.08^2}{0.4^2} \times 5$$

$$= 0.3 \text{ m}^3/\text{sec}$$

now; from Euler's model.

$$(Eu)_p = (Eu)_m$$

$$\left(\frac{8V^2}{\Delta P} \right)_p = \left(\frac{8V^2}{\Delta P} \right)_m$$

$$(\Delta P)_m = (\Delta P)_p \times \frac{V_m^2}{V_p^2} = 400 \times 5^2$$

$$= 10000 \text{ kPa}$$

Zillbhar

28. A spillway model is to be built to a geometrically similar scale of $\frac{1}{50}$ across a flume of 60cm width. The prototype is 15m high and the maximum head on it is expected to be 1.5m. Calculate

a) what height of the model and what head on the model should be used?

b) If the discharge passing the spillway for a particular head of the model is 12 lps. what flow per meter of the prototype is expected?

c) If the negative pressure in the model is 20cm, what is the negative pressure in the prototype? Is it possible?

→ Solution:

model:

$H_m = ?$

$h_{mp} = ?$

$Q_m = 12 \text{ lps}$

prototype:

$H_p = 15 \text{ m}$

$h_p = 1.5 \text{ m}$

$Q_p = ?$

$$a) \frac{H_p}{H_m} = 50 \Rightarrow H_m = \frac{15}{50} = 0.3 \text{ m}$$

$$\frac{h_p}{h_m} = 50 \Rightarrow h_m = \frac{1.5}{50} = 0.03 \text{ m}$$

$$L_r = 50$$

$$b) Q_p = Q_m \times (L_r)^{2.5}$$

$$\therefore Q_p = \frac{12}{1000} \times (50)^{2.5}$$

$$= 242.13 \text{ m}^3/\text{sec}$$

$$c) h_m = 20 \text{ cm}$$

$$\frac{h_m}{h_p} = \frac{1}{50}$$

$$\therefore h_p = 50 \times h_m$$

$$= 50 \times \frac{20}{1000}$$

$$= 1 \text{ m}$$

Bibha

22. A 1.25m diameter pipe has to be provided to convey oil of specific gravity 0.85 and kinematic viscosity 2.75×10^{-2} Stoke at $1.25 \text{ m}^3/\text{sec}$. In order to model the flow if a 120mm diameter pipe is used to convey water of kinematic viscosity of 0.01 Stoke, what should be the velocity and the discharge in the model?

Solution;

Model

$$d = 120 \text{ mm} = 0.12 \text{ m}$$

$$S = 1$$

$$\nu = 0.01 \text{ Stoke}$$

$$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}$$

$$Q_m = ?$$

$$V_m = ?$$

Prototype

$$d = 1.25 \text{ m}$$

$$S = 0.85$$

$$\nu = 2.75 \times 10^{-2} \text{ Stoke}$$

$$= 2.75 \times 10^{-4} \times 10^{-2} \text{ m}^2/\text{s}$$

$$= 2.75 \times 10^{-6} \text{ m}^2/\text{s}$$

$$Q_p = 1.25 \text{ m}^3/\text{s}$$

$$A_p = \pi \times \frac{1.25^2}{4} = 1.227 \text{ m}^2$$

$$V_p = \frac{Q_p}{A_p} = \frac{1.25}{1.227} = 1.0186 \text{ m/s}$$

Using;

$$(Re)_p = (Re)_m$$

$$\pi \left[\frac{VD}{\nu} \right]_p = \left[\frac{VD}{\nu} \right]_m$$

$$\text{or, } \frac{1.0186 \times 1.25}{2.75 \times 10^{-6}} = \frac{V_m \times 0.12}{0.01 \times 10^{-4}}$$

$$\therefore V_m = 3.858 \text{ m/s}$$

$$\frac{Q_p}{Q_m} = \frac{A_p}{A_m} \times \frac{V_p}{V_m} = \frac{D_p^2}{D_m^2} \times \frac{V_p}{V_m}$$

$$= \frac{1.25^2}{0.12^2} \times \frac{1.0186}{3.858}$$

$$= 28.648$$

$$\therefore Q_m = \frac{1.25}{28.648} = 0.0436 \text{ m}^3/\text{sec}$$

Zillbhar

24. An overflow- Structures 30m long discharges water at the rate of $750 \text{ m}^3/\text{s}$ under a head of 8m. A model test of this structure is to be carried out in a laboratory with a maximum supply available at 50 lps . Recommend a suitable scale ratio for the model.

Solution;

Model
 $Q_m = 50 \times 10^{-3} \text{ m}^3/\text{sec.}$

Prototype.
 $L_p = 30 \text{ m.}$
 $Q_p = 750 \text{ m}^3/\text{sec.}$
 $h_p = 8 \text{ m.}$

Using Froude's Law;

$$(Fr)_p = (Fr)_m.$$

$$\alpha, \left(\frac{V}{\sqrt{Lg}} \right)_p = \left(\frac{V}{\sqrt{Lg}} \right)_m.$$

$$\alpha, \frac{V_p^2}{L_p} = \frac{V_m^2}{L_m}$$

$$\Rightarrow \frac{V_p}{V_m} = \left(\frac{L_p}{L_m} \right)^{0.5}.$$

Also; $\frac{Q_p}{Q_m} = \left(\frac{L_p}{L_m} \right)^2 \times \frac{V_p}{V_m}$

$$\alpha, \frac{750}{0.05} = \left(\frac{L_p}{L_m} \right)^2 \times \left(\frac{L_p}{L_m} \right)^{0.5}$$

$$\alpha, \frac{750}{0.05} = \left(\frac{L_p}{L_m} \right)^{2.5}$$

$$\alpha, \frac{L_p}{L_m} = 46.82.$$

$$\therefore \frac{L_m}{L_p} = 0.02.$$

311bhan

27. The propeller of the ship is to be studied with 1:10 scale model.

a) Assuming that the propeller operates close to the surface, select the propeller speed of the model if the prototype speed is 600 rpm.

b) What torque would be expected if 1.2 Nm is measured on the model?

Solution;

Scale 1:10

$$\text{or, } \frac{L_m}{L_p} = \frac{1}{10}$$

Model

Prototype

$N_p = 600 \text{ rpm}$

$N_m = ?$

using Froude's law;

$$\text{a) } \left(\frac{V}{\sqrt{Lg}} \right)_p = \left(\frac{V}{\sqrt{Lg}} \right)_m$$

$$\text{or, } \frac{V_p^2}{L_p} = \frac{V_m^2}{L_m}$$

$$\text{or, } \frac{N_p^2 \times L_p^2}{L_p} = \frac{N_m^2 \times L_m^2}{L_m}$$

$$\text{or, } 600^2 = N_m^2 \times \frac{L_m}{L_p}$$

$$\text{or, } 600^2 \times 10 = N_m^2$$

$$\therefore N_m = 1897.36 \text{ rpm}$$

$$\text{b) } \frac{L_p F_p}{L_m F_m} = \frac{T_p}{T_m}$$

$$\text{or, } \frac{L_p}{L_m} \times \left(\frac{L_p}{L_m} \right)^3 = \frac{T_p}{T_m} \left[\because \frac{F_p}{F_m} = L^3 \right]$$

$$\therefore T_p = T_m \times L^4$$

$$= 1.2 \times 10^4$$

$$= 12000 \text{ Nm}$$

311bhar

Best Wishes!

Abhash Acharya

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